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Non-BPS D-Branes in Light-Cone Green-Schwarz Formalism

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Abstract

Non-BPS D-branes are difficult to describe covariantly in a manifestly supersymmetric formalism. For definiteness we concentrate on type IIB string theory in flat background in light-cone Green-Schwarz formalism. We study both the boundary state and the boundary conformal field theory descriptions of these D-branes with manifest $SO(8)$ covariance and go through various consistency checks. We analyze Sen's original construction of non-BPS D-branes given in terms of an orbifold boundary conformal field theory. We also directly study the relevant world-sheet theory by deriving the open string boundary condition from the covariant boundary state. Both these methods give the same open string spectrum which is consistent with the boundary state, as required by the world-sheet duality. The boundary condition found in the second method is given in terms of bi-local fields that are quadratic in Green-Schwarz fermions. We design a special "doubling trick" suitable to handle such boundary conditions and prescribe rules for computing all possible correlation functions without boundary insertions. This prescription has been tested by computing disk one-point functions of several classes of closed string states and comparing the results with the boundary state computation.

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1 Introduction and Summary

Although the Neveu-Schwarz-Ramond (NSR) formalism [1] is very advantageous for describing string theory in NS-NS backgrounds, it suffers from several drawbacks. These include lack of manifest space-time supersymmetry and difficulty in quantizing strings in RR backgrounds. Recent progress in Berkovits' pure spinor formalism [2] is encouraging in the direction of finding a tool free of such problems[1]. It is also desirable that we be able to describe all the valid objects in string theory in whatever language we find the

[1]See [3] for various extensions of the pure spinor formalism and other relevant works.

most convenient to work with. Non-BPS D-branes are certain non-supersymmetric non-perturbative solutions in string theory that are usually described in NSR formalism (see, for example, [4, 5, 6, 7, 8, 9, 10]). Unlike the BPS D-branes [11, 13] they are difficult to describe in a manifestly supersymmetric formalism where the basic world-sheet fields are space-time spinors [14]. In this paper we return to the same problem discussed in [14], namely: how to describe non-BPS D-branes in type II string theories in flat space using light-cone Green-Schwarz (GS) formalism? Although this is not a covariant description [15], it captures the basic problem of describing a non-BPS D-brane in a manifestly supersymmetric setup. A particular motivation to continue the above study is to look for the analogous D-branes in type IIB pp-wave background with RR-flux where the string quantization has been successfully done only in light-cone GS formalism [16]. This background is particularly interesting for exploring such questions in the context of AdS/CFT duality [18, 19].

In NSR formalism the world-sheet fermions are space-time vectors and satisfy simple linear boundary conditions for both the BPS and non-BPS D-branes. This is not true for non-BPS D-branes in GS formalism where the world-sheet fermions are also space-time fermions[2]. In spite of this difficulty non-BPS D-branes in flat background can be defined in a current algebraic framework [14] where the type II string theories in light-cone gauge are considered to be particular realizations of $SO(8)$ Kac-Moody algebra at level $k = 1$ [21]. Boundary condition on the local currents relevant to either a BPS or a non-BPS D-brane can be easily written down in terms of an automorphism which is simply the collection of reflections along the Neumann directions of the brane. This implies that for even dimensional world-volume this automorphism is "inner" whereas for odd dimensional world-volume this is "outer". It is the particular property of the fermionic variables used in GS formalism that it is difficult to describe "inner automorphism boundary conditions" in the realization of type IIA and "outer automorphism boundary conditions" in the realization of type IIB. These precisely correspond to the non-BPS D-branes in the two theories.

In current algebraic framework a non-BPS D-brane is given by a particular Ishibashi state. This is simply a sum over the left-right symmetric basis states in a particular highest weight representation with the outer automorphism (for type IIB) concerned acted on the

[2]The problem lies only in the fermionic sector of the world-sheet. Unless it is explicitly mentioned, the bosonic part will not play any role in any of our discussions.

right part [22, 23]. One way to see the problem with the type IIB non-BPS D-branes is that the action of the outer automorphism is not well defined on the right moving GS fermions, rather it is well defined only on the states spanning the particular highest weight representation involved [14]. Therefore to get an organized expression of the Ishibashi state in terms of the GS fermions a choice of the basis has to be made which spans only the relevant subspace. In [14] this basis was constructed explicitly in terms of the current modes. The problem with this choice is that the free module of the current modes contains null states which have to be removed by hand, a procedure which destroys manifest $SO(8)$ covariance. Indeed the analysis in [14] for the boundary states corresponding to branes with various orientations looks cumbersome. One has the similar problem also with the type IIA non-BPS D-branes.

Concentrating on the type IIB theory for definiteness we arrive at the covariant expression for a non-BPS boundary state by manipulating the Green-Schwarz basis states spanning the relevant subspace in a particular way. Similar expression was first extracted, using a nice trick, from the NSNS part of a BPS boundary state in [24] (see section 4 for comments on this method). Our result is very similar to but not exactly same as that found in [24]. This covariant expression appears to be much more complicated than that of a BPS D-brane [13]. However, it turns out that one can use a suitable bosonization and refermionization technique to construct a new anti-holomorphic spinor field $\bar{S}^a(\bar{z})$ out of the right-moving GS fermion $\tilde{S}^a(\bar{z})$ so that the non-BPS boundary state, when expressed in terms of the new oscillators $\bar{S}^a_n$, looks exactly like the NSNS part of a BPS D-instanton (zero world-volume dimension) boundary state in type IIB theory. There are certain computations for which this new form of the boundary state turns out to be very helpful. For example, as discussed in appendix D, one can derive the open string boundary condition from this form of the boundary state quite easily.

We also study the open string description of the system given in terms of a boundary conformal field theory (BCFT) and go through various consistency checks of the whole discussion of boundary state and BCFT. We partially discuss two approaches to the BCFT problem, both producing the same result for the open string spectrum[3]. This contains an R and an NS sector states. The R sector states have Bose-Fermi degeneracy at every level while the NS sector states, which include the tachyon, do not. We use this spectrum

[3]This spectrum is already known from the study in NSR formalism. Here we get the Green-Schwarz organization of the open string states.

and the covariant boundary state to check the open-closed duality on the world-sheet.

In the first approach we analyze the original method due to Sen [9] in GS formalism. In this method type IIB (IIA) string theory is regarded as type IIA (IIB) string theory with an orbifold projection by $(-1)^{\tilde{F}}$, where $\tilde{F}$ is the space-time fermion number contributed by the anti-holomorphic part of the world-sheet theory. To construct a non-BPS D-brane in type IIB one starts out with a suitable pair of BPS D-brane and anti-D-brane in type IIA and then projects out the whole configuration by $(-1)^{\tilde{F}}$. Since we are dealing with an orbifold BCFT and we know how to deal with the parent theory which involves only BPS D-branes one does not expect to see any obvious problem in computing the correlation functions. But we have not gone through the explicit analysis.

In the second approach, which is more direct, a non-BPS D-brane in the type IIB theory is described by the standard type IIB GS action with a certain boundary condition on the world-sheet fermions. Traditionally, the boundary condition corresponding to a D-brane is obtained by first varying the classical world-sheet action (with boundary) with respect to the basic fields and then setting the boundary term to zero. In the present context this method gives all the BPS boundary conditions quite easily, but it is hard to guess what a non-BPS boundary condition should be. However, the covariant form of the non-BPS boundary state and the reduction of its form to that of the NSNS part of a BPS D-instanton boundary state (as described above) enable us to derive the required open string boundary condition. A BPS boundary state satisfies a certain linear gluing condition relating the left and right moving oscillators [13]. From this it is easy to derive the quadratic gluing condition satisfied by the NSNS part of this boundary state[24]. This tells us what gluing condition a non-BPS boundary state should satisfy in terms of the S^a_n and $\bar{S}^a_n$ oscillators. Converting this gluing condition into the open string channel and using the bosonization and refermionization trick in the reverse direction we finally end up deriving the open string boundary condition in terms of the GS fermions $S^a(z)$ and $\tilde{S}^a(\bar{z})$. This boundary condition relates certain left and right moving fields which are bi-local and quadratic in GS fermions. Having obtained this boundary condition one can explain how that is compatible with the condition that one obtains by setting the boundary variation to zero.

It would be nice to be able to quantize the world-sheet theory in the "direct approach" described above. We have not tried to do this directly with such a bi-local boundary condition. Rather we have taken an approach where one comes up with a minimal set

of rules for computing all possible correlation functions of this boundary theory. In this paper we suggest that this is possible at least for the correlation functions without boundary insertions by explicitly prescribing the relevant rules. This involves extending the bi-local boundary conditions to all possible spin fields in the theory and designing a special "doubling trick" for handling such boundary conditions. Using our prescription we have computed disk one-point functions for several classes of closed string states with the puncture at the center of the disk. It has been shown that the results match, quite non-trivially, with the computation done with the covariant boundary state. The particular classes of closed string states have been chosen so as to verify certain numerical factors explicitly appearing in the covariant boundary state. The open string spectrum found in the "orbifold approach" discussed first can also be derived quite easily using the above doubling trick.

We have also made some naive attempts to look for the analogous D-branes in type IIB pp-wave background. In absence of a current algebraic framework we have tried to exploit the algebraic similarity between the string quantization in flat and pp-wave backgrounds. The algebraic structure of the zero modes is quite different in these two backgrounds and the attempts fail precisely because of some subtlety involving the fermionic zero modes.

The rest of the paper is organized as follows. Sec.2 contains the boundary state description. We first summarize the final result for the covariant boundary state, then discuss the bosonization and refermionization procedure to arrive at the simpler form. We discuss the BCFT description in sec.3 where the two approaches are presented in the two subsections 3.1 and 3.2. Some relevant points including some future directions are discussed in sec.4. Various technical details have been presented in several appendices.

2 Boundary State Description

For definiteness we shall consider type IIB string theory. All the D-branes that we shall consider here are instantonic in the sense that both the light-cone coordinates $(X^0 \pm X^9)$ satisfy Dirichlet boundary condition [12, 13]. The $(p+1)$-dimensional $(-1 \leq p \leq 7)$ world-volume of a Dp-brane is therefore euclidean. Without any loss of generality we shall always consider a Dp-brane (either BPS or non-BPS) to be aligned along coordinate directions such as $(x^{I_1}, \cdots, x^{I_{p+1}})$ for notational simplicity. Odd and even values of p correspond to BPS and non-BPS D-branes respectively in type IIB. In each case we

define a set of matrices giving the vector, spinor and conjugate spinor representations[4] of the set of reflections along the Neumann directions. For p odd, which corresponds to a BPS D-brane, we define,

$$\begin{aligned} M^V_{IJ} &= \lambda_I \delta_{IJ} \ , \quad \lambda_I = \begin{cases} 1 & \text{if } x^I \text{ is a Dirichlet direction} \ , \\ -1 & \text{if } x^I \text{ is a Neumann direction} \ , \end{cases} \\ M^S_{ab} &= \left(\gamma^{I_1}\bar{\gamma}^{I_2}\gamma^{I_3}\bar{\gamma}^{I_4}\cdots\gamma^{I_p}\bar{\gamma}^{I_{p+1}}\right)_{ab} \ , \\ M^C_{\dot{a}\dot{b}} &= \left(\bar{\gamma}^{I_1}\gamma^{I_2}\bar{\gamma}^{I_3}\gamma^{I_4}\cdots\bar{\gamma}^{I_p}\gamma^{I_{p+1}}\right)_{\dot{a}\dot{b}} \ , \end{aligned} \tag{2.1}$$

where the gamma matrices in the last two equations are placed with a chosen ordering of the Neumann indices. These expressions correspond to the following block off diagonal form of the 16-dimensional gamma matrices: $\Gamma^I = \begin{pmatrix} 0 & \gamma^I_{a\dot{b}} \\ \bar{\gamma}^I_{\dot{a}b} & 0 \end{pmatrix}$. Unless it is explicitly mentioned otherwise we shall always consider a real representation (as in [15]), in which case $\bar{\gamma}^I = (\gamma^I)^{\mathrm{T}}$. For p even, corresponding to a non-BPS D-brane, we denote these matrices by $\bar{M}^V_{IJ}$, $\bar{M}^S_{a\dot{b}}$ and $\bar{M}^C_{\dot{a}b}$ respectively and use the similar definitions. All the M matrices defined this way are orthogonal. The fermionic gluing condition satisfied by the boundary state of a BPS D-brane is simply given by [13],

$$(S^a_n + i\eta M^S_{ab}\tilde{S}^b_{-n})|\mathrm{BPS}, p, \eta\rangle = 0 \ , \quad \forall n \in \mathbb{Z}. \tag{2.2}$$

But notice that because of the particular index structure of $\bar{M}^S_{a\dot{b}}$ the boundary state for a non-BPS D-brane does not satisfy such a simple gluing condition. Although the above gluing condition does not have a simple generalization to the non-BPS case it was shown in [14] that the current algebraic construction of the BPS boundary states does have. This method has been reviewed in appendix A. It turns out that a covariant expression for the non-BPS boundary state can be obtained by manipulating the Green-Schwarz basis expansion in an Ishibashi state in a certain way (see appendix B). Similar expression was first extracted, using a nice trick, from the NSNS part of a BPS boundary state in [24]. The result reported here is very similar to but not exactly same as that found in [24]. Here we shall summarize the final result. The boundary state of a non-BPS D-brane in type-IIB situated at the origin of the transverse directions is given by,

$$|\,\mathrm{B\!\!\!/PS}\,\rangle = \bar{\mathcal{N}}_p \int dk_\perp \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \alpha^I_{-n} \bar{M}^V_{IJ} \tilde{\alpha}^J_{-n}\right) |e\rangle\rangle \otimes |k_\perp\rangle \ , \tag{2.3}$$

[4]Recall that because of the triality symmetry [15] of the associated Dynkin diagram there are three lowest (8) dimensional representations of $SO(8)$.

where $\bar{\mathcal{N}}_p$ is a normalization constant[5]. $k_\perp$ is the momentum in the transverse space which includes x^0 and x^9. As explained in appendix B, $|e\rangle\rangle$ is a current algebra Ishibashi state. Its covariant expression is given by, with some additional prescription that we shall spell out shortly,

$$
\begin{aligned}
|e\rangle\rangle &= |\bar{M}^V, \mathrm{even}\rangle + |\bar{M}^V, \mathrm{odd}\rangle \ , \\
|\bar{M}^V \mathrm{even}\rangle &= \cosh\left(\sqrt{X_{\bar{M}^V}}\right) \bar{M}^V_{IJ} |I\rangle \otimes \widetilde{|J\rangle} \ , \\
|\bar{M}^V, \mathrm{odd}\rangle &= -\frac{\sinh\left(\sqrt{X_{\bar{M}^V}}\right)}{\sqrt{X_{\bar{M}^V}}} Y^{\dot{a}\dot{b}}_{\bar{M}^V} |\dot{a}\rangle \otimes \widetilde{|\dot{b}\rangle} \ .
\end{aligned}
\tag{2.4}
$$

The operators $X_{\bar{M}^V}$ and $Y^{\dot{a}\dot{b}}_{\bar{M}^V}$ are given by,

$$
\begin{aligned}
X_{\bar{M}^V} = \sum_{m,n>0} \Bigg[& \frac{1}{8} J_{-m,-n} \tilde{J}_{-m,-n} + \frac{1}{16} \sum_{IJ} \bar{\lambda}_{\{IJ\}} J^{IJ}_{-m,-n} \tilde{J}^{IJ}_{-m,-n} + \\
& \frac{2}{384} \sum_{\{IJKL\} \in \mathcal{K}} \bar{\lambda}_{\{IJKL\}} J^{IJKL}_{-m,-n} \tilde{J}^{IJKL}_{-m,-n} \Bigg] \ ,
\end{aligned}
\tag{2.5}
$$

$$
Y^{\dot{a}\dot{b}}_{\bar{M}^V} = \sum_{n>0} \left[\frac{1}{8} \sum_I \bar{\lambda}_I \gamma^I_{a\dot{a}} \gamma^I_{b\dot{b}} S^a_{-n} \tilde{S}^b_{-n} + \frac{1}{48} \sum_{IJK} \bar{\lambda}_{\{IJK\}} \gamma^{IJK}_{a\dot{a}} \gamma^{IJK}_{b\dot{b}} S^a_{-n} \tilde{S}^b_{-n} \right] \ , \tag{2.6}
$$

where we have denoted the eigenvalues of $\bar{M}^V$ by $\bar{\lambda}_I$ and used the notation: $\bar{\lambda}_{\{IJ\cdots\}} = \bar{\lambda}_I \bar{\lambda}_J \cdots$. We also used the following definitions [24],

$$
J_{mn} = S^a_m S^a_n, \quad J^{IJ}_{mn} = \gamma^{IJ}_{ab} S^a_m S^b_n \ , \quad J^{IJKL}_{mn} = \gamma^{IJKL}_{ab} S^a_m S^b_n \ , \tag{2.7}
$$

and similarly for the the right moving variables. A multi-indexed gamma matrix is anti-symmetric under interchange of any of the two indices and it is defined in the following way,

$$
\gamma^{I_1 I_2 \cdots I_n} = \begin{cases} \gamma^{I_1} \bar{\gamma}^{I_2} \cdots \ , & I_1 \neq I_2 \neq \cdots \neq I_n \ , \\ 0 \ , & \text{otherwise} \end{cases} \ . \tag{2.8}
$$

Although the last two equations in (2.4) have been given compact forms in terms of the cosh and sinh operators they are actually understood as series expansions where only integer powers of $X_{\bar{M}^V}$ appear. The overall sign of the state $|\bar{M}^V, \mathrm{odd}\rangle$ in (2.4) depends on a certain convention while that of the state $|\bar{M}^V, \mathrm{even}\rangle$ does not. Our convention is to take $\tilde{S}^a(\bar{z})$ to commute with the left moving vector spin field $\psi^I(z)$ and anti-commute with the conjugate spinor spin field $S^{\dot{a}}(z)$[6]. Finally, let us explain the last term on the

[5]We shall show in appendix C that the boundary state (2.3) satisfies the open-closed world-sheet duality for the expected value of $\bar{\mathcal{N}}_p$.

[6]This convention is just opposite to that followed in [14].

right hand side of eq.(2.5). We divide the set of all possible sets of four indices $\{I, J, K, L\}$ into two subsets of equal order: $\mathcal{K}$ and $\mathcal{K_D}$ such that for every element $\{I, J, K, L\}$ in $\mathcal{K}$ there is a dual set of indices $\{M, N, O, P\}$ in $\mathcal{K_D}$ satisfying $\epsilon^{IJKLMNOP} \neq 0$. Since J^{IJKL}_{mn} are self dual operators,

$$J^{IJKL}_{mn} = \frac{1}{4!}\epsilon^{IJKLMNOP} J^{MNOP}_{mn} \ , \tag{2.9}$$

the number of independent components is same as the order of $\mathcal{K}$ or $\mathcal{K_D}$ modulo the anti-symmetrization among the four indices. The summation in the last term on the right hand side of eq.(2.5) is restricted to the elements of $\mathcal{K}$. Notice that $\mathcal{K}$ is not unique, it has to be chosen by hand and the state $|e\rangle\rangle$ does depend on this choice. We would like to point out that this choice is at the very level of basis construction in GS formalism. The GS variables corresponding to different choices are related in a complicated manner through bosonization and refermionization[7]. To see this more clearly and to spell out the additional implicit prescription in eq.(2.4) through an example, let us concentrate on the states at level 3. This is the minimum level where the operators $J^{IJKL}_{m,n}$ appear in the basis construction. In NSR formalism the rank 5 tensor states at this level are given by $\psi^I_{-1/2}\psi^J_{-1/2}\psi^K_{-1/2}\psi^L_{-1/2}\psi^M_{-3/2}|0\rangle_{NS}$. There is no self duality condition on the first four indices and there are 560 independent components. In GS formalism all these states have to come from two sets of states: $\gamma^{IJKL}_{ab}S^a_{-1}S^b_{-2}|M\rangle$ and $\gamma^{IJ}_{ab}\gamma^{KLM}_{c\dot{d}}S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$, each contributing 280 independent states[8] making up the total of 560. Therefore to specify the basis one has to choose 280 states from each of the two groups. This choice, for the group of states $\gamma^{IJKL}_{ab}S^a_{-1}S^b_{-2}|M\rangle$, is given by $\mathcal{K}$. States for which $\{I, J, K, L\} \in \mathcal{K}_D$ should come from the other group: $\gamma^{IJ}_{ab}\gamma^{KLM}_{c\dot{d}}S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$. Notice that the two groups of states appear in $|\bar{M}^V, \text{even}\rangle$ and $|\bar{M}^V, \text{odd}\rangle$ respectively in eq.(2.4). Reduction of the first group of states to the independent ones (modulo anti-symmetrization) as mentioned above is taken care of explicitly by the constrained sum in the last term in eq.(2.5). As can be seen

[7] I would like to thank A. Sen for emphasizing this point.

[8] The tensor index structure of $\gamma^{IJ}_{ab}\gamma^{KLM}_{c\dot{d}}S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$ does not quite lead to 280 independent states. The counting is more complicated because of the antisymmetry among all the three oscillators. But there is an indirect way: These states, along with the state $\gamma^{IJ}_{ab}\gamma^{K}_{c\dot{d}}S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$ make up all the states in $S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$ which can be counted to have 448 independent states. One can count the number of independent rank 3 tensor states from NSR formalism without any trouble. This also turns out to be 448. In addition to $\gamma^{IJ}_{ab}\gamma^{K}_{c\dot{d}}S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$ they also include $\gamma^{IJ}_{ab}S^a_{-1}S^b_{-2}|K\rangle$ and $\gamma^{IJK}_{a\dot{b}}S^a_{-3}|\dot{b}\rangle$ whose counting (in GS formalism) gives 224 and 56 respectively. This gives (448-224-56)=168 for $\gamma^{IJ}_{ab}\gamma^{K}_{c\dot{d}}S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$. Therefore we need 280 for $\gamma^{IJ}_{ab}\gamma^{KLM}_{c\dot{d}}S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$ to make up 448 for $S^a_{-1}S^b_{-1}S^c_{-1}|\dot{d}\rangle$.

easily by inspection that this is not true for the second group of states. This reduction has to be done by hand in the proper terms. Once it is done, one ends up having the rank 5 tensor states, with the same properties of the NSR states mentioned above, with free sum over all the indices in the expansion of the boundary state. Once this procedure is followed at all the levels, the boundary state, when expanded in terms of the basis states, does not depend on $\mathcal{K}$. This is guaranteed, as the boundary state written in this fashion should take the same form as that obtained in light-cone NSR formalism (though it is difficult to prove this in GS formalism). It is only when the basis states are written in terms of the GS variables, the $\mathcal{K}$ dependence comes in.

Bosonization and Refermionization

We shall now show that the complicated expression of $|e\rangle\rangle$ in eq.(2.4) can be given a much simpler form in terms of suitably defined variables which are obtained by bosonizing and refermionizing (say) the antiholomorphic triad $\left\{\tilde{S}^a(\bar{z}), \tilde{S}^{\dot{a}}(\bar{z}), \tilde{\psi}^I(\bar{z})\right\}$ on the full plane in a certain manner. Let us first define a new spinor in the following way,

$$\hat{S}^{\dot{a}}(\bar{z}) = \bar{M}^C_{\dot{a}a}\tilde{S}^a(\bar{z}) \ . \tag{2.10}$$

Using the following relations that can be derived from the definitions of the $\bar{M}$ matrices given below eq.(2.1),

$$\begin{aligned} \left(\bar{M}^S\right)^{\mathrm{T}} &= (-1)^{p/2}\bar{M}^C \ , \quad \text{(recall } p \text{ is even)} \ , \\ \bar{M}^C\gamma^I\left(\bar{M}^S\right)^{\mathrm{T}} &= \bar{\lambda}_I\bar{\gamma}^I \ , \end{aligned} \tag{2.11}$$

one can show,

$$\hat{S}^{\dot{a}}(\bar{z})\bar{\gamma}^{IJ\cdots}_{\dot{a}\dot{b}}\hat{S}^{\dot{b}}(\bar{w}) = \bar{\lambda}_{\{IJ\cdots\}}\tilde{S}^a(\bar{z})\gamma^{IJ\cdots}_{ab}\tilde{S}^b(\bar{w}) \ , \tag{2.12}$$

where we mean to have a product of even number of gamma matrices as indicated by the index structure. The multi-indexed $\bar{\gamma}$ matrices are defined by replacing γ's by $\bar{\gamma}$'s and vice versa in eq.(2.8),

$$\bar{\gamma}^{I_1I_2\cdots I_n} = \begin{cases} \bar{\gamma}^{I_1}\gamma^{I_2}\cdots \ , & I_1 \neq I_2 \neq \cdots \neq I_n \ , \\ 0 \ , & \text{otherwise} \ . \end{cases} \tag{2.13}$$

Notice that although $\hat{S}^{\dot{a}}\bar{\gamma}^{IJKL}_{\dot{a}\dot{b}}\hat{S}^{\dot{b}}$ is an anti-self-dual operator (as can be easily verified by using the last equation in (G.14)) while $\tilde{S}^a(\bar{z})\gamma^{IJKL}_{ab}\tilde{S}^b(\bar{w})$ is self-dual, eq.(2.12) holds

because of the following relation,

$$\bar{\lambda}_{\{IJKL\}} = -\bar{\lambda}_{\{MNOP\}} \ , \quad \text{whenever } \epsilon^{IJKLMNOP} \neq 0 \ . \tag{2.14}$$

We now suitably bosonize and refermionize $\hat{S}^{\dot{a}}(\bar{z})$ to obtain a new fermion $\bar{S}^a(\bar{z})$ such that,

$$\begin{aligned}
\bar{S}^a(\bar{z})\bar{S}^a(\bar{w}) &= \hat{S}^{\dot{a}}(\bar{z})\hat{S}^{\dot{a}}(\bar{w}) \ , \\
\bar{S}^a(\bar{z})\gamma^{IJ}_{ab}\bar{S}^b(\bar{w}) &= \hat{S}^{\dot{a}}(\bar{z})\bar{\gamma}^{IJ}_{\dot{a}\dot{b}}\hat{S}^{\dot{b}}(\bar{w}) \ , \\
\bar{S}^a(\bar{z})\gamma^{IJKL}_{ab}\bar{S}^b(\bar{w}) &= \begin{cases} \hat{S}^{\dot{a}}(\bar{z})\bar{\gamma}^{IJKL}_{\dot{a}\dot{b}}\hat{S}^{\dot{b}}(\bar{w}) & \text{for } \{I,J,K,L\} \in \mathcal{K} \ , \\ -\hat{S}^{\dot{a}}(\bar{z})\bar{\gamma}^{IJKL}_{\dot{a}\dot{b}}\hat{S}^{\dot{b}}(\bar{w}) & \text{for } \{I,J,K,L\} \in \mathcal{K}_{\mathcal{D}} \ . \end{cases}
\end{aligned} \tag{2.15}$$

Denoting the spin fields of $\bar{S}^a(\bar{z})$ by $\bar{\psi}^I(\bar{z})$ and $\bar{S}^{\dot{a}}(\bar{z})$ one can also relate them to the spin fields of $\tilde{S}^a(\bar{z})$,

$$\begin{aligned}
\bar{\psi}^I(\bar{z}) &= \bar{\lambda}_I\tilde{\psi}^I(\bar{z}) \ , \\
\bar{S}^{\dot{a}}(\bar{z})\bar{\gamma}^{IJ\cdots}_{\dot{a}\dot{b}}\bar{S}^{\dot{b}}(\bar{w}) &= \bar{\lambda}_{\{IJ\cdots\}}\tilde{S}^{\dot{a}}(\bar{z})\bar{\gamma}^{IJ\cdots}_{\dot{a}\dot{b}}\tilde{S}^{\dot{b}}(\bar{w}) \ , \\
\bar{S}^a(\bar{z})\gamma^{IJ\cdots}_{a\dot{a}}\bar{S}^{\dot{a}}(\bar{w}) &= \bar{\lambda}_{\{IJ\cdots\}}\tilde{S}^a(\bar{z})\gamma^{IJK}_{a\dot{a}}\tilde{S}^{\dot{a}}(\bar{w}) \ ,
\end{aligned} \tag{2.16}$$

where the second and third equations have products of even and odd number of gamma matrices respectively as indicated by the index structures. In case of four vector indices (I,J,K,L) the second equation is understood to be true for $\{I,J,K,L\} \in \mathcal{K}$. For $\{I,J,K,L\} \in \mathcal{K}_{\mathcal{D}}$ the equation will have an additional factor of (-1). Equations (2.12), (2.15) and (2.16) clearly show that $\left\{\bar{S}^a(\bar{z}), \bar{S}^{\dot{a}}(\bar{z}), \bar{\psi}^I(\bar{z})\right\}$ corresponds to the same gamma matrix representation in the frame transformed by $\bar{M}^V$ as that corresponding to $\left\{\tilde{S}^a(\bar{z}), \tilde{S}^{\dot{a}}(\bar{z}), \tilde{\psi}^I(\bar{z})\right\}$ in the untransformed frame. Therefore, for the zero modes $\bar{S}^a_0$, one has,

$$\bar{S}^a_0|\bar{I}\rangle = \frac{1}{\sqrt{2}}\gamma^I_{a\dot{a}}|\bar{\dot{a}}\rangle \ , \tag{2.17}$$

where $|\bar{I}\rangle$ and $|\bar{\dot{a}}\rangle$ are the ground states created by the spin fields $\bar{\psi}^I(\bar{z})$ and $\bar{S}^{\dot{a}}(\bar{z})$ respectively.

From the above discussion, it is not difficult to convince oneself that the NS-sector states (following to the nomenclature of NSR formalism[9]) on the anti-holomorphic side written in terms of the oscillators $\tilde{S}^a_n$'s and the states $|\widetilde{I}\rangle$ and $|\widetilde{\dot{a}}\rangle$ can be translated, without

[9]According to the notation in eq.3.1, these states span the irreducible representation $\Pi^{(e)}$.

much trouble, in terms of the oscillators $\bar{S}^a_n$'s and the states $|\bar{I}\rangle$ and $|\bar{\dot{a}}\rangle$. This allows us to rewrite the state $|e\rangle\rangle$ in eq.(2.4) in terms of these new variables. In fact, using eq.(2.14) it is not difficult to see that the translated expression should simply take the same form as in eq.(2.4) with $\bar{M}^V$ set to identity. One then uses the algebraic manipulation of appendix B in the reverse direction to finally arrive at the following expression,

$$|e\rangle\rangle = \frac{1}{2}\left(|\mathrm{BPS},-1,+\rangle_{\bar{S}} + |\mathrm{BPS},-1,-\rangle_{\bar{S}}\right) \ , \tag{2.18}$$

where $|\mathrm{BPS},-1,\eta\rangle_{\bar{S}}$ ($\eta = \pm 1$) is obtained, up to a normalization constant, by replacing: $\tilde{S}^a_{-n} \to \bar{S}^a_{-n}$, $\widetilde{|I\rangle} \to |\bar{I}\rangle$ and $\widetilde{|\dot{a}\rangle} \to |\bar{\dot{a}}\rangle$ in the fermionic part of a BPS D-instanton (zero world-volume dimension) boundary state.

$$|\mathrm{BPS},-1,\eta\rangle_{\bar{S}} = \exp\left(-i\eta\sum_{n>0,a} S^a_{-n}\bar{S}^a_{-n}\right)\left[\sum_I |I\rangle\otimes|\bar{I}\rangle - i\eta\sum_{\dot{a}}|\dot{a}\rangle\otimes|\bar{\dot{a}}\rangle\right] \ , \tag{2.19}$$

It is interesting to notice that $|e\rangle\rangle$ in eq.(2.18) takes the form of the NSNS part of the BPS D-instanton boundary state. Certain analysis, such as those in appendices C and D, can be enormously simplified with this form.

We shall test the covariant form in (2.4) by checking the open-closed duality on the cylinder. We shall also explicitly verify the numerical factors 1/8, 1/16, 2/384 and 1/48 appearing in eqs.(2.5, 2.6) by computing one-point functions of the relevant classes of closed string states in a boundary conformal field theory description.

3 Boundary Conformal Field Theory Description

In this section we shall study the problem from the open string point of view. We have discussed two approaches. The first one (subsec.3.1) contains the analysis of Sen's original orbifold method in GS formalism. In the second approach (subsec.3.2) we study the problem more directly in terms of a BCFT with certain boundary condition. Both the methods produce the same open string spectrum which is consistent, according to the world-sheet duality, with the covariant boundary state (appendix C).

3.1 Orbifold Approach

Type IIB (IIA) superstring theory can be regarded as the $(-1)^{\tilde{F}}$ orbifold of type IIA (IIB) theory. Here $\tilde{F}$ is the space-time fermion number contributed by the right moving sector of

the world-sheet. The original construction of type IIB (IIA) non-BPS D-branes due to Sen went through as follows: Take a non-supersymmetric configuration of pair of coincident BPS D-brane and anti-D-brane in type IIA (IIB) and then project the configuration out by $(-1)^{\tilde{F}}$. This gives the corresponding non-BPS D-brane in type IIB (IIA). Here we shall outline the same procedure in GS formalism. As an outcome of this analysis we find the Green-Schwarz organization of the open string spectrum on a non-BPS D-brane. The same result is also found in the second approach in subsec.3.2.

Type IIB Theory as $(-1)^{\tilde{F}}$ orbifold of Type IIA Theory

Since the bosonic part is same in both the two theories and the projection does not act on this we shall focus only on the fermionic part of the theory. In light-cone gauge, this part of type IIA theory can be viewed as the following particular realization of $\widehat{SO}(8)_{k=1}$[21, 14]:

$$\text{type IIA} : \left(\Pi^{(e)} \oplus \Pi^{(\bar{\delta})}\right) \otimes \left(\Pi^{(e)} \oplus \Pi^{(\delta)}\right) \ , \tag{3.1}$$

where $\Pi^{(w)}$ is the irreducible representation corresponding to the highest weight w. e, δ and $\bar{\delta}$ are the $SO(8)$ fundamental weights corresponding to the vector, spinor and conjugate spinor representations respectively. In terms of the GS oscillators a generic state in type IIA, therefore, takes the following form,

$$S^{a_1}_{-n_1} S^{a_2}_{-n_2} \cdots S^{a_N}_{-n_N} \tilde{S}^{\dot{a}_1}_{-m_1} \tilde{S}^{\dot{a}_2}_{-m_2} \cdots \tilde{S}^{\dot{a}_M}_{-m_M} \begin{pmatrix} |I\rangle \\ |\dot{a}\rangle \end{pmatrix} \otimes \begin{pmatrix} \widetilde{|J\rangle} \\ \widetilde{|a\rangle} \end{pmatrix} \ . \tag{3.2}$$

Obviously, one needs to impose the left-right level-matching condition on it. While $(-1)^{\tilde{F}}$ acts as an identity operator on the left-moving variables, its action on the right-moving variables is given by,

$$(-1)^{\tilde{F}} : \quad \tilde{S}^{\dot{a}}(\bar{z}) \to -\tilde{S}^{\dot{a}}(\bar{z}) \ , \quad \widetilde{|I\rangle} \to \widetilde{|I\rangle} \ , \quad \widetilde{|a\rangle} \to -\widetilde{|a\rangle} \ . \tag{3.3}$$

Therefore $\tilde{S}^{\dot{a}}(\bar{z})$ is an NS fermion in the twisted sector. The NS ground state $\widetilde{|0\rangle}$ is taken to be odd under $(-1)^{\tilde{F}}$. Otherwise the set of invariant twisted sector states reduces to null after imposing the left-right level-matching condition. The allowed states in the orbifolded theory are given by,

$$\text{untwisted} \quad : \quad \left\{ \text{any number of } S^a_{-n}\text{'s} \begin{pmatrix} |I\rangle \\ |\dot{a}\rangle \end{pmatrix} \right\} \otimes \left\{ \begin{array}{c} \text{even number of } \tilde{S}^{\dot{a}}_{-n}\text{'s}\widetilde{|I\rangle} \\ \\ \text{odd number of } \tilde{S}^{\dot{a}}_{-n}\text{'s}\widetilde{|a\rangle} \end{array} \right\} ,$$

$$\text{twisted} \quad : \quad \left\{\text{any number of } S^a_{-n}\text{'s} \begin{pmatrix} |I\rangle \\ |\dot{a}\rangle \end{pmatrix}\right\} \otimes \left\{\text{odd number of } \tilde{S}^{\dot{a}}_{-r}\text{'s}|\widetilde{0}\rangle\right\} \ . \tag{3.4}$$

It can be shown that,

$$\left\{ \begin{array}{c} \text{even number of } \tilde{S}^{\dot{a}}_{-n}\text{'s}|\widetilde{I}\rangle \\ \\ \text{odd number of } \tilde{S}^{\dot{a}}_{-n}\text{'s}|\widetilde{\dot{a}}\rangle \end{array} \right\} \to \Pi^{(e)} \ , \quad \left\{\text{odd number of } \tilde{S}^{\dot{a}}_{-r}\text{'s}|\widetilde{0}\rangle\right\} \to \Pi^{(\bar{\delta})} \ , \tag{3.5}$$

implying that the spectrum in the orbifolded theory is same as that of the type IIB theory as shown in eq.(A.1).

Open String Spectrum

Let us now consider a pair of BPS D-brane and anti-D-brane in type IIA. There are four sectors of open strings on the D$\bar{\text{D}}$ system with different CP factors Λ^q ($q = 1, \cdots, 4$). The various CP factors and the corresponding open string boundary conditions are give by,

$$\begin{aligned}
\Lambda^1 &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} : \qquad S^a(\tau,\sigma) = \bar{M}^S_{a\dot{a}}\tilde{S}^{\dot{a}}(\tau,\sigma) \ , \qquad \text{at } \sigma = 0, \pi \ , \\
\Lambda^2 &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} : \qquad S^a(\tau,\sigma) = -\bar{M}^S_{a\dot{a}}\tilde{S}^{\dot{a}}(\tau,\sigma) \ , \quad \text{at } \sigma = 0, \pi \ , \\
\Lambda^3 &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} : \qquad S^a(\tau,0) = \bar{M}^S_{a\dot{a}}\tilde{S}^{\dot{a}}(\tau,0) \ , \qquad S^a(\tau,\pi) = -\bar{M}^S_{a\dot{a}}\tilde{S}^{\dot{a}}(\tau,\pi) \ , \\
\Lambda^4 &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} : \qquad S^a(\tau,0) = -\bar{M}^S_{a\dot{a}}\tilde{S}^{\dot{a}}(\tau,0) \ , \quad S^a(\tau,\pi) = \bar{M}^S_{a\dot{a}}\tilde{S}^{\dot{a}}(\tau,\pi) \ .
\end{aligned} \tag{3.6}$$

Recall that the matrices $\bar{M}^V$, $\bar{M}^S$ and $\bar{M}^C$ correspond to a BPS D-brane in the type IIA theory. The above boundary conditions show that the open strings with CP factors Λ^1 and Λ^2 have integer modding (R sector) and that with CP factors Λ^3 and Λ^4 have half integer modding (NS sector). Under the action of $(-1)^{\tilde{F}}$ the above boundary conditions change following (3.3). It is easy to see that this action can alternatively be viewed as an action performed only on the CP factors.

$$(-1)^{\tilde{F}} : \left\{ \begin{array}{ll} \Lambda^1 & \leftrightarrow \Lambda^2 \ , \\ \Lambda^3 & \leftrightarrow \Lambda^4 \ . \end{array} \right. \tag{3.7}$$

Therefore only the R sector states with CP factor $\dfrac{\mathbb{1}_2}{\sqrt{2}}$ and NS sector states with CP factor $\dfrac{\sigma^1}{\sqrt{2}}$ survive the projection. Here $\mathbb{1}_2$ and σ^1 are the two dimensional identity matrix and

the first Pauli matrix respectively. Therefore the fermionic part of all possible open string states living on the non-BPS D-brane of type IIB are given by,

$$\begin{aligned} |\{N_{an}\}, I\rangle \otimes \frac{\mathbb{1}_2}{\sqrt{2}} &= \left(\prod_{a,n} (S^a_{-n})^{N_{an}}\right) |I\rangle \otimes \frac{\mathbb{1}_2}{\sqrt{2}} , \\ |\{N_{an}\}, \dot{a}\rangle \otimes \frac{\mathbb{1}_2}{\sqrt{2}} &= \left(\prod_{a,n} (S^a_{-n})^{N_{an}}\right) |\dot{a}\rangle \otimes \frac{\mathbb{1}_2}{\sqrt{2}} , \\ |\{N_{ar}\}\rangle \otimes \frac{\sigma^1}{\sqrt{2}} &= \left(\prod_{a,r} (S^a_{-r})^{N_{ar}}\right) |0\rangle \otimes \frac{\sigma^1}{\sqrt{2}} , \end{aligned} \tag{3.8}$$

where $n = 1, 2, \cdots$, $r = 1/2, 3/2, \cdots$, $N_{an}, N_{ar} = 0, 1$ and the product of oscillators has an implicit fixed ordering. Every state of the above kind has to be multiplied with the bosonic part $|\{N_{In}\}, \vec{p}\rangle$ to give the complete state. Here $N_{In} = 0, 1, 2, \cdots$ is the eigenvalue of the number operator $\alpha^I_{-n}\alpha^I_n$ and $\vec{p}$ is the open string momentum along the D-brane world-volume. Notice that in the R sector we have equal number of bosons and fermions at every mass level. This spectrum is same as that of open strings living on a BPS D-brane of type IIA. Presence of the NS sector states makes the spectrum non-supersymmetric. In particular at the massless level one has $|I\rangle \otimes \mathbb{1}_2/\sqrt{2}$ and $|\dot{a}\rangle \otimes \mathbb{1}_2/\sqrt{2}$ from the R sector. Additional fermionic states $|a\rangle \otimes \sigma^1/\sqrt{2} = S^a_{-1/2}|0\rangle \otimes \sigma^1/\sqrt{2}$ come, at this level, from the NS sector showing that the number of massless fermions is just double compared to that on a BPS D-brane. This is precisely the case for a non-BPS D-brane[6, 7, 25]. Also it is the NS sector which gives the tachyon $|0\rangle \otimes \sigma^1/\sqrt{2}$.

The whole configuration of a non-BPS D-brane in type IIB can be further projected out by $(-1)^{\tilde{F}}$ of the present theory. In the bulk it takes us to type IIA theory. On the boundary, this projection is expected to remove the NS sector states giving the supersymmetric open string spectrum on a BPS D-brane in type IIA. This can possibly be shown following the arguments similar to those given in [9], but we have not explicitly gone through that analysis.

3.2 Direct Approach

Although in the above method the open string theory is simply given by an orbifold BCFT whose parent BCFT is solved and therefore potentially does not have any obvious problem, it is somewhat indirect. Instead of coming from type IIA theory through the process of orbifolding, here we shall study the BCFT describing a non-BPS D-brane directly in type IIB. In this approach the relevant BCFT should be given by the usual

type IIB Green-Schwarz action on the world-sheet with a certain open string boundary condition. In the case of BPS D-branes these boundary conditions can be easily obtained by setting the boundary term in the variation of the classical action to zero. This can not be done so easily for the non-BPS D-branes[10]. However, the covariant form of the boundary state (2.4) allows one to derive certain quadratic gluing conditions[11] which can, in turn, be converted into open string channel to obtain the required boundary condition. The details of this analysis is given in appendix D. The final result for the open string boundary condition on the upper half plane (UHP) is given by,

$$S^a(z)S^b(w) \;=\; \mathcal{M}^{ab}_{cd}\,\tilde{S}^c(\bar{z})\tilde{S}^d(\bar{w})\;, \qquad \text{at } z=\bar{z},\; w=\bar{w}\;. \tag{3.9}$$

where,

$$\mathcal{M}^{ab}_{cd} \;=\; \frac{1}{8}\delta_{ab}\delta_{cd}+\frac{1}{16}\sum_{I,J}\bar{\lambda}_{\{IJ\}}\gamma^{IJ}_{ab}\gamma^{IJ}_{cd}+\frac{2}{384}\sum_{\{I,J,K,L\}\in\mathcal{K}}\bar{\lambda}_{\{IJKL\}}\gamma^{IJKL}_{ab}\gamma^{IJKL}_{cd}\;. \tag{3.10}$$

Equation (3.9) has to be interpreted to relate normal ordered operators when the arguments coincide. Let us now explain how the above boundary condition is compatible with the condition that one gets by setting the boundary term in the variation of the action to zero. In the present case this procedure gives the following condition,

$$S^a(z)\delta S^a(z)=\tilde{S}^b(\bar{z})\delta\tilde{S}^b(\bar{z})\;, \quad \text{at } z=\bar{z}. \tag{3.11}$$

Contracting the indices on the left hand side of (3.9) and using some of the trace formulas in (G.14) one finds,

$$S^a(z)S^a(w)=\tilde{S}^b(\bar{z})\tilde{S}^b(\bar{w})\;, \quad \text{at } z=\bar{z}, w=\bar{w}. \tag{3.12}$$

The fact that we have bi-local boundary condition enables us to take variation of the fields at different points on the boundary independently. We first take the variation of

[10]The current algebra gluing condition (A.2) immediately gives an open string boundary condition in terms of the local $SO(8)$ currents. One can then try to study the boundary theory in the current algebraic language. This formalism requires the additional information of what kind of highest weight representations are realized in the theory. Also as we have seen in the boundary state analysis in [14], manifest covariance is lost in the current algebraic language. Our problem here is to find a covariant description in terms of the GS fermions.

[11]Similar gluing conditions, which do not exactly match with the results obtained here (see eq.(D.1)), were found before in [24].

the fields at $w = \bar{w}$ in equation (3.12). Then after normal ordering the operators we take the limit: $w \to z$. This gives the condition (3.11) with normal ordering. Notice that it is difficult to guess the boundary condition (3.9) from the requirement of (3.11).

Let us now discuss the boundary conditions for the spin fields. For the vector spin fields this can be readily written down,

$$\psi^I(z) = \bar{M}^V_{IM} \tilde{\psi}^M(\bar{z}) \ , \quad \text{at } z = \bar{z} \ . \tag{3.13}$$

Since the conjugate spinor spin field $S^{\dot{a}}(z)$ appears in the OPE of $S^a(z)$ with $\psi^I(z)$ (and similarly for the anti-holomorphic side) any bulk correlation function can be reduced to a form where only $S^a(z)$, $\tilde{S}^a(\bar{z})$, $\psi^I(z)$ and $\tilde{\psi}^I(\bar{z})$ appear. Therefore the boundary conditions (3.9, 3.13) are sufficient to compute any such correlation function. Nevertheless, one can also directly write down the boundary conditions involving $S^{\dot{a}}(z)$ and $\tilde{S}^{\dot{a}}(\bar{z})$. Using the Fiertz identity (G.12) one can decompose $S^{\dot{a}}(z)S^{\dot{b}}(w)$ into antisymmetric tensors of even ranks. Similarly the identity (G.13) enables us to break $S^a(z)S^{\dot{b}}(w)$ into antisymmetric tensors of odd ranks. Boundary conditions for these antisymmetric tensors will then involve the appropriate twisting given by the matrix $\bar{M}^V_{IJ}$. The final results can be summarized as,

$$\left.\begin{aligned} S^{\dot{a}}(z)S^{\dot{b}}(w) &= \mathcal{M}^{\dot{a}\dot{b}}_{\dot{c}\dot{d}}\ \tilde{S}^{\dot{c}}(\bar{z})\tilde{S}^{\dot{d}}(\bar{w}) \ , \\ S^{a}(z)S^{\dot{b}}(w) &= \mathcal{M}^{a\dot{b}}_{c\dot{d}}\ \tilde{S}^{c}(\bar{z})\tilde{S}^{\dot{d}}(\bar{w}) \ , \end{aligned}\right\} \quad \text{at } z = \bar{z},\ w = \bar{w} \ , \tag{3.14}$$

where,

$$\begin{aligned} \mathcal{M}^{\dot{a}\dot{b}}_{\dot{c}\dot{d}} &= \frac{1}{8}\delta_{\dot{a}\dot{b}}\delta_{\dot{c}\dot{d}} + \frac{1}{16}\sum_{IJ}\bar{\lambda}_{\{IJ\}}\bar{\gamma}^{IJ}_{\dot{a}\dot{b}}\bar{\gamma}^{IJ}_{\dot{c}\dot{d}} + \frac{2}{384}\sum_{\{I,J,K,L\}\in\mathcal{K}}\bar{\lambda}_{\{IJKL\}}\bar{\gamma}^{IJKL}_{\dot{a}\dot{b}}\bar{\gamma}^{IJKL}_{\dot{c}\dot{d}} \ , \\ \mathcal{M}^{a\dot{b}}_{c\dot{d}} &= \frac{1}{8}\sum_{I}\bar{\lambda}_I\gamma^I_{a\dot{b}}\gamma^I_{c\dot{d}} + \frac{1}{48}\sum_{IJK}\bar{\lambda}_{\{IJK\}}\gamma^{IJK}_{a\dot{b}}\gamma^{IJK}_{c\dot{d}} \ . \end{aligned} \tag{3.15}$$

The above boundary conditions look complicated and we have not tried to directly quantize the corresponding world-sheet theory. Instead we take an approach where the solution is given in terms of a minimal set of rules for computing all possible correlation functions in the theory. In the following we design a special "doubling trick" and use that to compute the open string spectrum and prescribe a set of rules for computing all possible correlation functions without boundary insertions leaving the rest of the correlators for future work.

3.2.1 Boundary Spectrum and Prescription for Computing Correlation Functions without Boundary Insertions

In the BCFT describing a BPS D-brane the fundamental world-sheet fields have linear boundary conditions. In that case one can use the usual doubling trick [26] to reduce a correlation function of bulk operators on the UHP to a correlation function on the full plane which is simply a computation on the holomorphic side of the bulk theory without boundary. As we have seen above, in case of a non-BPS D-brane the world-sheet fields that are space-time spinors have bi-local boundary conditions. In this case we generalize the above-mentioned trick to achieve the same thing.

The local properties of the world-sheet fields are same both in the boundary and the bulk theory. Let us consider a set of three holomorphic fields $\left\{\mathcal{S}^a(z), \mathcal{S}^{\dot{a}}(z), \Psi^I(z)\right\}$ with the implied $SO(8)$ transformation properties living on the full complex plane which have the same local properties as the holomorphic triad $\left\{S^a(z), S^{\dot{a}}(z), \psi^I(z)\right\}$ of the bulk theory. As usual, we define $\Psi^I(z)$ to be,

$$\Psi^I(u) = \begin{cases} \psi^I(z)|_{z=u} \ , & \Im u \geq 0 \ , \\ \\ \bar{\lambda}_I \psi^I(\bar{z})|_{\bar{z}=u} \ , & \Im u \leq 0 \ . \end{cases} \tag{3.16}$$

We actually intend to consider bi-local operators constructed out of $\mathcal{S}^a(z)$ and $\mathcal{S}^{\dot{a}}(z)$ rather than the individual ones. These are given by,

$$\mathcal{S}^a(u) \cdots \mathcal{S}^b(v) = \begin{cases} S^a(z) \cdots S^b(w)|_{z=u, w=v} \ , & \Im u, \Im v \geq 0 \ , \\ \\ \mathcal{M}^{ab}_{cd} \ \tilde{S}^c(\bar{z}) \cdots \tilde{S}^d(\bar{w})|_{\bar{z}=u, \bar{w}=v} \ , & \Im u, \Im v \leq 0 \ , \end{cases} \tag{3.17}$$

$$\mathcal{S}^{\dot{a}}(u) \cdots \mathcal{S}^{\dot{b}}(v) = \begin{cases} S^{\dot{a}}(z) \cdots S^{\dot{b}}(w)|_{z=u, w=v} \ , & \Im u, \Im v \geq 0 \ , \\ \\ \mathcal{M}^{\dot{a}\dot{b}}_{\dot{c}\dot{d}} \ \tilde{S}^{\dot{c}}(\bar{z}) \cdots \tilde{S}^{\dot{d}}(\bar{w})|_{\bar{z}=u, \bar{w}=v} \ , & \Im u, \Im v \leq 0 \ , \end{cases} \tag{3.18}$$

$$\mathcal{S}^a(u) \cdots \mathcal{S}^{\dot{b}}(v) = \begin{cases} S^a(z) \cdots S^{\dot{b}}(w)|_{z=u, w=v} \ , & \Im u, \Im v \geq 0 \ , \\ \\ \mathcal{M}^{a\dot{b}}_{c\dot{d}} \ \tilde{S}^c(\bar{z}) \cdots \tilde{S}^{\dot{d}}(\bar{w})|_{\bar{z}=u, \bar{w}=v} \ , & \Im u, \Im v \leq 0 \ , \end{cases} \tag{3.19}$$

The dots imply that the above relations are understood to be true even when other operators appear in between in a correlation function. Notice also that both the arguments

	$S^a(z)$	$\tilde{S}^a(\bar{z})$	$\psi^I(z)$	$\tilde{\psi}^I(\bar{z})$	$S^{\dot{a}}(z)$	$\tilde{S}^{\dot{a}}(\bar{z})$
F	1	0	0	0	1	0
$\tilde{F}$	0	1	0	0	0	1

Table 1: Assignment of fermion numbers to various bulk fields

u and v are either in the upper or the lower half plane. This implies that we need to consider only a sub-sector of all possible holomorphic correlation functions of $\mathcal{S}^a(z)$, $\mathcal{S}^{\dot{a}}(z)$ and $\Psi^I(z)$ on the full plane.

Having defined the essential fields on the full plane, we can now derive the boundary spectrum quite easily[12]. Using eq.(3.17) and the boundary condition (3.9) one can show that on the cylinder,

$$\begin{aligned} \mathcal{S}^a(\tau, 2\pi)\mathcal{S}^b(\tau', 2\pi) &= \mathcal{S}^a(\tau, 0)\mathcal{S}^b(\tau', 0) \ , \\ \text{i.e. } \mathcal{S}^a(\tau, 2\pi) &= \pm\mathcal{S}^a(\tau, 0) \ , \end{aligned} \tag{3.20}$$

which says that there is an R and an NS sectors of states in the boundary spectrum in agreement with the result found in subsec.3.1.

Assigning the fermion numbers F and $\tilde{F}$, defined only mod 2, for the left and right-moving fields as shown in table 1, we now give our prescription for computing a generic correlation function of the bulk insertions on UHP.

1. In order for the correlation function to be nonzero the net global fermion charges of all the closed string insertions should be $F_C = \tilde{F}_C = 0$. This implies that if m, n, p, q, r, s are the number of $S^a(z)$'s, $\tilde{S}^a(\bar{z})$'s, $S^{\dot{a}}(z)$'s, $\tilde{S}^{\dot{a}}(\bar{z})$'s, $\psi^I(z)$'s and $\tilde{\psi}^I(\bar{z})$'s respectively, then the correlator is nonzero only when

$$m + p = \text{even} \ , \quad n + q = \text{even} \ , \quad r, s \text{ arbitrary} \ . \tag{3.21}$$

2. A correlation function satisfying the conditions in (3.21) can be computed with the following prescription.

    - Pair up all the left and right moving fields that are space-time spinors such that all of them can be written in terms of the bilinear of $\mathcal{S}^a(z)$ and $\mathcal{S}^{\dot{a}}(w)$ by

[12] I thank A. Sinha and N. V. Suryanarayana for discussion on this point.

using eqs.(3.17, 3.18, 3.19). Do-ability of this is guaranteed by the condition (3.21)[13].

- Using the usual doubling trick write all the left and right moving fields that are space-time vector in terms of $\Psi^I(z)$'s.
- Performing the above two steps we end up having a holomorphic correlation function on the full plane. Compute this following the same rules for computing the holomorphic correlation functions of the bulk theory without boundary.

The second rule described above is simply a generalization of the usual doubling trick to the present case. It is difficult to compute the correlators which do not satisfy condition (3.21). Although we do not have a direct proof of the first rule, it gives the expected result that a non-BPS D-brane does not source RR closed string states. Notice that the corresponding statement for a BPS D-brane is that a correlator without boundary insertions is nonzero only when $F_C + \tilde{F}_C = 0$.

3.2.2 Closed String One-point Functions

Here we shall demonstrate through explicit computations how to use the above prescription to find bulk correlators. In particular we shall compute several closed string one-point functions on the unit disk and show that when the vertex operator is inserted at the center of the disk the result matches with that obtained through boundary state computation.

We shall verify the following relation for certain closed string states $|\Phi\rangle$,

$$\langle\langle e|\Phi\rangle = C \lim_{\zeta,\bar{\zeta}\to 0} \langle \Phi(\zeta,\bar{\zeta})\rangle_D \ , \tag{3.22}$$

where $\Phi(\zeta,\bar{\zeta})$ is the vertex operator for the state $|\Phi\rangle$ with $(\zeta,\bar{\zeta})$ being the complex coordinate system on the unit disk. $\langle\cdots\rangle_D$ denotes a disk correlation function in the BCFT of fermions that we are presently considering. C is an overall constant that does not depend on which closed string state we choose. The states for which we verify relation (3.22) are,

$$|\Phi^{IJ}\rangle \ = \ |I\rangle \otimes \widetilde{|J\rangle} \ ,$$

[13]Generically this can be done in more than one ways. Consistency would require all of them to give the same final result. Although we expect that it should be possible to establish this using various trace formulas in (G.14), we have not found a complete proof.

$$
\begin{aligned}
|\Phi^{IJ}_{m,n}\rangle &= -\gamma^I_{a\dot a}\gamma^J_{b\dot b}S^a_{-m}\tilde S^b_{-n}|\dot a\rangle\otimes\widetilde{|\dot b\rangle}\ ,\\
|\Phi^{(IJK),(MNP)}_{m,n}\rangle &= -\gamma^{IJK}_{a\dot a}\gamma^{MNP}_{b\dot b}S^a_{-m}\tilde S^b_{-n}|\dot a\rangle\otimes\widetilde{|\dot b\rangle}\ ,\\
|\Phi^{IJ}_{m,n,p,q}\rangle &= J_{-m,-n}\tilde J_{-p,-q}|I\rangle\otimes\widetilde{|J\rangle}\ ,\\
|\Phi^{(MN),(M'N'),IJ}_{m,n,p,q}\rangle &= J^{MN}_{-m,-n}\tilde J^{M'N'}_{-p,-q}|I\rangle\otimes\widetilde{|J\rangle}\ ,\\
|\Phi^{(MNPQ),(M'N'P'Q'),IJ}_{m,n,p,q}\rangle &= J^{MNPQ}_{-m,-n}\tilde J^{M'N'P'Q'}_{-p,-q}|I\rangle\otimes\widetilde{|J\rangle}\ .
\end{aligned}
\tag{3.23}
$$

These classes of states have been chosen to explicitly verify the numerical factors 1/8, 1/16, 2/384 and 1/48 appearing in eqs.(2.5, 2.6). The above states are antisymmetric under interchanges of the vector indices that are kept inside $(\cdots)$. To demonstrate the computation, here we shall verify eq.(3.22) only for the first two classes of states in the above list leaving the analysis for the rest in appendix E. The vertex operators for the states $|\Phi^{IJ}\rangle$ and $|\Phi^{IJ}_{m,n}\rangle$ are,

$$
\Phi^{IJ}(\zeta,\bar\zeta)=\psi^I(\zeta)\tilde\psi^J(\bar\zeta)\ ,\quad \Phi^{IJ}_{m,n}(\zeta,\bar\zeta)=-\gamma^I_{a\dot a}\gamma^J_{b\dot b}\Phi^{ab\dot a\dot b}_{m,n}(\zeta,\bar\zeta)\ ,
\tag{3.24}
$$

respectively, where,

$$
\Phi^{ab\dot a\dot b}_{m,n}(\zeta,\bar\zeta) = -\oint_\zeta\frac{d\zeta_1}{2\pi i}\oint_{\bar\zeta}\frac{d\bar\zeta_2}{2\pi i}(\zeta_1-\zeta)^{-m-1/2}(\bar\zeta_2-\bar\zeta)^{-n-1/2}S^a(\zeta_1)\tilde S^b(\bar\zeta_2)S^{\dot a}(\zeta)\tilde S^{\dot b}(\bar\zeta)\ .
\tag{3.25}
$$

Defining the bra state: $\langle\Phi|=\langle\phi_R|\otimes\langle\phi_L|$ corresponding to a ket $|\Phi\rangle=|\phi_L\rangle\otimes|\phi_R\rangle$, where $\langle\phi_{R/L}|$ has the conjugated oscillators placed just in the reverse order of that in $|\phi_{R/L}\rangle$, we get the following inner products,

$$
\langle\Phi^{IJ}|\Phi^{I'J'}\rangle=\delta_{I,I'}\ \delta_{J,J'}\ ,\quad \langle\Phi^{IJ}_{m,n}|\Phi^{I'J'}_{m',n'}\rangle=64\delta_{m,m'}\ \delta_{n,n'}\ \delta_{I,I'}\ \delta_{J,J'}\ .
\tag{3.26}
$$

States belonging to different classes in eqs.(3.23) can be shown to be mutually orthogonal. Using these inner products and eq.(2.4) we get the following results,

$$
\langle\langle e|\Phi^{IJ}\rangle=\bar\lambda_I\delta_{IJ}\ ,\quad \langle\langle e|\Phi^{IJ}_{m,n}\rangle=8\delta_{m,n}\ \bar\lambda_I\delta_{IJ}\ .
\tag{3.27}
$$

We shall now compute the right hand side of eq.(3.22) for the vertex operators in eqs.(3.24)

in the BCFT using the prescriptions in 3.2.1. Using the following conformal transformation which relates the unit disk $(\zeta, \bar{\zeta})$ to UHP $(z, \bar{z})$,

$$z(\zeta) = i\frac{1+\zeta}{1-\zeta} \ , \tag{3.28}$$

one can relate correlation functions of bulk primaries on unit disk and on UHP in the following way,

$$\langle \prod_i \Phi_i(\zeta_i, \bar{\zeta}_i) \rangle_D \ = \ \prod_i \left((z_i')^{h_i} (\bar{z}_i')^{\bar{h}_i} \right) \langle \prod_i \Phi_i(z_i, \bar{z}_i) \rangle_{UHP} \ , \tag{3.29}$$

where Φ_i is a bulk primary with conformal weight $(h_i, \bar{h}_i)$, $z_i = z(\zeta_i)$ and $z_i' = z'(\zeta_i)$ with the prime denoting the first derivative. Using this and the standard doubling trick for the vertex $\Phi^{IJ}(\zeta, \bar{\zeta})$ one gets,

$$\begin{aligned} \langle \Phi^{IJ}(0,0) \rangle_D \ &= \ 2\langle \psi^I(i) \tilde{\psi}^J(-i) \rangle_{UHP} \ , \\ &= \ 2\bar{\lambda}_J \langle \Psi^I(i) \Psi^J(-i) \rangle \ , \end{aligned} \tag{3.30}$$

where $\langle \cdots \rangle$ refers to a holomorphic correlation function on the full plane. Finally using the OPE: $\Psi^I(z)\Psi^J(w) \sim \frac{\delta_{IJ}}{(z-w)}$ one gets,

$$\langle \Phi^{IJ}(0,0) \rangle_D = \bar{\lambda}_I \delta_{IJ} \ . \tag{3.31}$$

Comparing this with the first equation of (3.27) one fixes the constant:

$$C = 1 \ . \tag{3.32}$$

We then turn to the computation of $\langle \Phi^{IJ}_{m,n}(0,0) \rangle_D$. Using (3.29) we may write,

$$\langle \Phi^{IJ}_{m,n}(0,0) \rangle_D = -\oint_i \frac{dz_1}{2\pi i} \oint_{-i} \frac{d\bar{z}_2}{2\pi i} \mathcal{J}_{m,n}(z_1, \bar{z}_2) \gamma^I_{a\dot{a}} \gamma^J_{b\dot{b}} \langle S^a(z_1) \tilde{S}^b(\bar{z}_2) S^{\dot{a}}(i) \tilde{S}^{\dot{b}}(-i) \rangle_{UHP} \ , \tag{3.33}$$

where,

$$\mathcal{J}_{m,n}(z_1, \bar{z}_2) = 4(z_1 - i)^{-m-1/2}(z_1 + i)^{m-1/2}(\bar{z}_2 + i)^{-n-1/2}(\bar{z}_2 - i)^{n-1/2} \ . \tag{3.34}$$

Using the generalized doubling trick described in the previous subsection we now reduce the right hand side of eq.(3.33) to a correlation function on the full plane,

$$\langle \Phi^{IJ}_{m,n}(0,0) \rangle_D = \bar{\lambda}_J \oint_i \frac{dz}{2\pi i} \oint_{-i} \frac{dw}{2\pi i} \mathcal{J}_{m,n}(z, w) \gamma^I_{a\dot{a}} \gamma^J_{b\dot{b}} \langle \mathcal{S}^a(z) \mathcal{S}^b(w) \mathcal{S}^{\dot{a}}(i) \mathcal{S}^{\dot{b}}(-i) \rangle \ . \tag{3.35}$$

Notice that there is an overall sign difference between the eqs. (3.33) and (3.35). This is simply because the second integral in eq.(3.33) has been converted to a holomorphic integral in eq.(3.35) which reverses the sense of the contour. The correlation function in the above integrand and all others that are needed for the computations done in appendix E have been evaluated in appendix F. Using eq.(F.1) and the result for the relevant integral given in eq.(H.15) one gets,

$$\langle \Phi^{IJ}_{m,n}(0,0)\rangle_D = 8\delta_{mn}\bar{\lambda}_I\delta_{IJ} \ . \tag{3.36}$$

With $C = 1$ and the second equation in (3.27) the above result verifies (3.22) for the states $|\Phi^{IJ}_{m,n}\rangle$.

4 Discussion

Here we discuss a few points that are relevant to the present work including some future directions.

- **Method of Ref.[24]**

  It was suggested in [24] that the covariant expression for a non-BPS boundary state can be obtained by going through the following two steps:

  1. Write the NSNS part of a BPS boundary state (whose covariant expression is known) in a form where spinor and conjugate spinor matrices M^S and M^C respectively do not appear at all, rather only the vector representation matrix M^V appears.
  2. Replace M^V, which corresponds to a "BPS automorphism" by $\bar{M}^V$, the one corresponding to a "non-BPS automorphism", in the final expression obtained in the first step.

  This method is expected to work because of the following reason. Notice that being $SO(8)$ tensors the current modes are glued with the vector representation matrix M^V or $\bar{M}^V$, depending on whether the relevant D-brane is BPS or non-BPS respectively (see eqs.(A.2, A.4)). Therefore it should be possible to write the coefficient of any term in the basis expansion of the Ishibashi state purely in terms of the vector representation matrix. This, along with the fact that both the NSNS

part of a BPS boundary state and a non-BPS boundary state belong to the same subspace ($\Pi^{(e)} \otimes \Pi^{(e)}$) of the full closed string Hilbert space [14], guarantees the doability of the first step. Once the first step is achieved the second step is analogous to computing a function for a different argument.

This method works well provided a particular technical subtlety is taken into account. After completing the first step if one naively goes through the replacement in the second step then one finds that the coefficients of a class of basis states in the expansion of the boundary state become zero which were previously nonzero. Similar situation arises in the current algebraic derivation given in appendix B as well. Although the present method by itself does not say anything special about this situation, we have argued in appendix B (see discussion below eq.(B.6)) that this is unexpected. To arrive at the expression (2.4) one has to take care of this subtlety in a way similar to that prescribed in appendix B. Recall that it is the expression (2.4) which passes through the open-closed duality check in appendix C.

- **Comments of the computation of open-point functions**

  As mentioned below eq.(2.8), the overall sign of the state $|\bar{M}^V, \text{odd}\rangle$ in eqs.(2.4) is convention dependent. Had we followed the convention of [14] (see footnote 6) this sign would have been (+). Notice that the sign of the state $|\bar{M}^V, \text{even}\rangle$ does not depend on the convention. One may wonder how to see this ambiguity in the computation of the one-point functions on the BCFT side. In the BCFT method we have reduced the computation of the one-point function of a closed string state to computing certain complex integrals where a holomorphic correlation function on the full plane appears in the integrand. Because of the presence of the spin fields the results of these correlation functions involve various branch cuts on the complex plane. In order to compute the integrals the correlation functions have to be computed for certain ranges of values of the arguments. Typically this procedure is ambiguous as one has to make choice of the branches. It turns out that this feature affects our final result (by introducing a sign ambiguity) only for the second and third classes of the states in the list (3.23), which are precisely the ones relevant to the state $|\bar{M}^V, \text{odd}\rangle$. This can be seen in the following way: The contours for the required integrals have been shown in fig.1(A). This can be computed by taking the limit: $u \to i,\ v \to -i$ on the contours in fig.1(B). The integrals relevant to

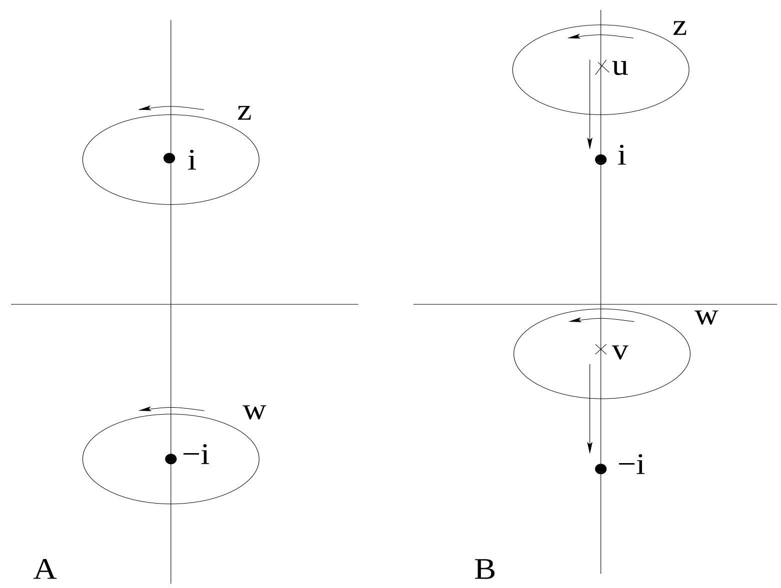

Figure 1: (A) Contours for computing the integrals relevant to the one-point functions of the second and third classes of states in the list (3.23). (B) A possible limiting procedure to compute the integrals.

fig.1(B) requires one to compute the relevant correlators, as given by eqs.(F.1, F.2), in a region where $|u| > |w|$. This, in turn, requires us to pass $\mathcal{S}^b(w)$ through $\mathcal{S}^{\dot{a}}(u)$. This procedure contributes a factor of $(\pm i)$, as evident from the square root brunch cut in $(w-u)$ present in the results (F.15, F.15). It can be checked that the other part of the integrand, namely $\mathcal{J}_{mn}(z,w,u,v)$ (such that $\mathcal{J}_{mn}(z,w,i,-i) = \mathcal{J}_{mn}(z,w)$ which is given in eq.(3.34)) does not have a brunch cut in $(w-u)$. After taking the required limit all the brunch cuts go away by absorbing the factor of i, but leaving a sign ambiguity. Notice that this sign ambiguity does not arise for the other correlators in (F.3, F.4, F.5) which are relevant for computing one-point functions of some of the states in $|\bar{M}^V, \text{even}\rangle$. It can be checked from the results given in eqs.(F.16, F.17, F.18, F.19, F.20, F.21) that no extra phase arises while passing z_2 and w_2 simultaneously through u.

- **More on BCFT**

  Both the approaches in sec.3 have been given partial description. We have not discussed computation of the correlation functions in the "orbifold approach". Al-

though this is an orbifold BCFT whose parent theory is solved and one does not see any obvious problem in computing correlators, it may be useful to understand the details of the computations. We have prescribed rules for computing the correlators without boundary insertions in the "direct approach". One might get clues on how to set up rules for computing correlators with boundary insertions by going through those computations in the "orbifold approach".

- **Trial in pp-wave**

  We have given some naive attempts to find the analogous non-BPS D-barnes in the type IIB pp-wave background. There is a lot of algebraic similarity between string quantization in this and in the flat background [16]. In fact the fermionic parts of the boundary states of class I (according to the classification of [29]) D-branes in pp-wave[27, 28] look exactly same as that of the BPS D-branes in flat space (see, for example, eqs.(2.13) and (2.14) in [28]). Given this one may wonder if a class of non-BPS boundary states in pp-wave can be obtained by taking the fermionic parts to be exactly same as those (eqs.(2.4)) in the flat background. This is also precisely what one would do if one generalizes the method of [24], as described above, to pp-wave background. It turns out that one can use the same bosonization and refermionization technique of sec.2 to compute the necessary overlap of the boundary sate to evaluate the cylinder diagram. This is found not to have an open string interpretation unless the BPS condition is satisfied, namely $|r - s| = 2$, where r and s are the number of Neumann directions in first and last four coordinate directions $(x^1, \cdots, x^4)$ and $(x^5, \cdots, x^8)$ respectively. Clearly this can not be satisfied for the case that we are interested in, namely $r + s =$ odd.

  One may try another generalization from the flat space to pp-wave in the following way. We have shown in sec.2 that any non-BPS boundary state (fermionic part) in flat space can be given by the NSNS part of a BPS D-instanton boundary state written in terms of the oscillators of $S^a(z)$ and $\bar{S}^a(\bar{z})$ (see eqs.(2.10, 2.12, 2.15)). The information about the dimensionality and alignment of the brane are completely absorbed into the definition of $\bar{S}^a(\bar{z})$. Therefore if the NSNS part of a BPS D-instanton boundary sate (eqs.(2.15)-(2.17) of [29]) written in terms of the oscillators of $S^a(z)$ and $\bar{S}^a(\bar{z})$ admits an open string interpretation in pp-wave then we are done. The answer turns out to be "no".

All these attempts are at a very naive level where one tries to exploit some algebraic features in the string quantization. Notice that the algebraic structure of the string zero modes in pp-wave is quite different from that in flat space. In fact the failure in both the above computations is actually caused by the fermion zero modes. It was pointed out in [28] that given a form of a boundary state, these zero modes put strong restriction on the dimensionality and alignment of the D-brane. Indeed the boundary states have been classified in [29] on the basis of dimensionality and alignment of D-branes. The reason why the above methods do not work may be that we are trying to enforce a particular form of the boundary state for a D-brane of different dimensionality. It might very well happen that the non-BPS D-branes that we are looking for do exist with some completely different forms of the boundary states. If possible, it will be very useful to have a current algebraic formulation where this question may be asked more clearly.

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A Current Algebraic Construction

While the perturbative excitation spectrum (only the world-sheet fermionic part) of type IIA theory is given by eq.(3.1), the same for type IIB theory can be shown to be,

$$\text{type IIB}: \quad (\Pi^{(e)} \oplus \Pi^{(\bar{\delta})}) \otimes (\Pi^{(e)} \oplus \Pi^{(\bar{\delta})}) \ . \tag{A.1}$$

The above notation has been explained below eq.(3.1). It was argued in [14] that the fermionic part $|e\rangle\rangle$ of the non-BPS boundary state in eq.(2.3) is the Ishibashi state corresponding to the highest weight e and it satisfies the following gluing condition,

$$\left(J_n^{IJ} + \tau.\tilde{J}_{-n}^{IJ}\right)|e\rangle\rangle = 0 \ , \quad \forall n \in \mathbb{Z} \ , \tag{A.2}$$

where J_n^{IJ} and $\tilde{J}_n^{IJ}$ are the modes of the local left and right moving $SO(8)$ currents respectively constructed out of the world-sheet fermions.

$$J_n^{IJ} = \frac{i}{4}\left(\gamma^I \bar{\gamma}^J\right)_{ab} \sum_{m \in \mathbb{Z}} : S_m^a S_{n-m}^b : \ , \qquad \forall n \in \mathbb{Z} \ , \tag{A.3}$$

(similarly for the right moving current modes) where :: denotes the usual oscillator normal ordering. τ is the outer automorphism given by[14],

$$\tau.\tilde{J}_n^{IJ} = \bar{M}_{IK}^V \bar{M}_{JL}^V \tilde{J}_n^{KL} \ . \tag{A.4}$$

The Ishibashi state $|e\rangle\rangle$ is given by,

$$|e\rangle\rangle = \sum_N |N, e\rangle \otimes \mathcal{T}\Theta|\widetilde{N, e}\rangle, \tag{A.5}$$

where $|N, e\rangle$ is a complete set of orthonormal basis vectors in $\Pi^{(e)}$:

$$\{|N, e\rangle\} \equiv \left\{ \left[\prod_{a,n>0} \left(S_{-n}^a\right)^{N_{an}} |I\rangle\right]_{\sum N_{an}=even} \ , \quad \left[\prod_{a,n>0} \left(S_{-n}^a\right)^{N_{an}} |\dot{a}\rangle\right]_{\sum N_{an}=odd} \right\} \ , \tag{A.6}$$

where $N_{an} = 0, 1$ and the products are ordered products with some chosen ordering. $\mathcal{T}$ and Θ are two Hilbert space operators which act only on the right moving part of a state. It was argued in [14] that the action of Θ on these basis states is actually trivial, i.e.

$$\Theta|\widetilde{N, e}\rangle = |\widetilde{N, e}\rangle \ . \tag{A.7}$$

All the states in (A.6) can be obtained by applying current creation operators on the highest weight state $|e\rangle = (|I = 1\rangle - i|I = 2\rangle)/\sqrt{2}$. The action of $\mathcal{T}$ on all such states can be obtained from the following,

$$\begin{aligned} \mathcal{T}\tilde{J}_n^{IJ}\mathcal{T}^{-1} &= \tau.\tilde{J}_n^{IJ} = \bar{M}_{IK}^V \bar{M}_{JL}^V \tilde{J}_n^{KL} \ , \\ \mathcal{T}|\widetilde{I}\rangle &= \bar{M}_{IJ}^V |\widetilde{J}\rangle \ , \end{aligned} \tag{A.8}$$

Although we have a nice action of $\mathcal{T}$ on the current oscillators, being an outer automorphism, $\mathcal{T}$ does not have a well-defined action on the $\tilde{S}_n^a$ oscillators. This is the main obstruction against finding a simple covariant expression for $|e\rangle\rangle$.

[14]These outer automorphisms which correspond to reflections along odd number of directions take the vector representation to itself but switches between the spinor and conjugate spinor representations. This is why only the Ishibashi state $|e\rangle\rangle$ is realized in a non-BPS boundary state. Notice that in the BPS case the automorphisms involved are inner as they correspond to reflections along even number of directions. In this case all the representations go to themselves. As a result both the Ishibashi states $|e\rangle\rangle$ and $|\bar{\delta}\rangle\rangle$ are realized forming the NSNS and the RR parts of the boundary state respectively [14].

B Derivation of the Covariant Form

Despite the problem mentioned below eq.(A.8), the covariant expression for non-BPS boundary state given in eqs.(2.4, 2.5, 2.6) can be derived in the current algebraic framework by manipulating the basis states (A.6) in a certain manner. Taking the expression of $|e\rangle\rangle$ given in (A.5) and using the basis states (A.6) one gets,

$$
\begin{aligned}
|e\rangle\rangle &= |\text{even}\rangle + |\text{odd}\rangle \ , \\
|\text{even}\rangle &= \frac{1}{2}\mathcal{T} \sum_{\{N_{an}\},I} \left(1 + (-1)^{\sum_{a,n} N_{an}}\right) |\{N_{an}\}, I\rangle \ , \\
|\text{odd}\rangle &= \frac{1}{2}\mathcal{T} \sum_{\{N_{an}\},\dot{a}} \left(1 - (-1)^{\sum_{a,n} N_{an}}\right) |\{N_{an}\}, \dot{a}\rangle \ ,
\end{aligned} \tag{B.1}
$$

where,

$$
\begin{aligned}
|\{N_{an}\}, I\rangle &= \prod_{n>0,a} \left(S^a_{-n}\right)^{N_{an}} |I\rangle \otimes \prod_{n>0,a} \left(\tilde{S}^a_{-n}\right)^{N_{an}} \widetilde{|I\rangle} \ , \\
&= \prod_{n>0,a} \left(iS^a_{-n}\tilde{S}^a_{-n}\right)^{N_{an}} |I\rangle \otimes \widetilde{|I\rangle} \ .
\end{aligned} \tag{B.2}
$$

$$
\begin{aligned}
|\{N_{an}\}, \dot{a}\rangle &= \prod_{n>0,a} \left(S^a_{-n}\right)^{N_{an}} |\dot{a}\rangle \otimes \prod_{n>0,a} \left(\tilde{S}^a_{-n}\right)^{N_{an}} \widetilde{|\dot{a}\rangle} \ , \\
&= i(-1)^{1+\sum_{a,n} N_{an}} \prod_{n>0,a} \left(iS^a_{-n}\tilde{S}^a_{-n}\right)^{N_{an}} |\dot{a}\rangle \otimes \widetilde{|\dot{a}\rangle} \ .
\end{aligned} \tag{B.3}
$$

Notice that while going from the first step to the second in eq. (B.3) we need to keep in mind that each $\tilde{S}^a_{-n}$ oscillator picks up a $(-)$ sign when it passes through the state $|\dot{a}\rangle$. Plugging in the result (B.2) into the second equation of (B.1) one can write,

$$
\begin{aligned}
|\text{even}\rangle &= \mathcal{T} \cos\left(\sum_{n>0,a} S^a_{-n}\tilde{S}^a_{-n}\right) |I\rangle \otimes \widetilde{|I\rangle} \ , \\
&= \mathcal{T} \cosh\left(\sqrt{X}\right) |I\rangle \otimes \widetilde{|I\rangle} \ ,
\end{aligned} \tag{B.4}
$$

where,

$$
\begin{aligned}
X &= \sum_{m,n>0} S^a_{-m} S^b_{-n} \tilde{S}^a_{-m} \tilde{S}^b_{-n} \ , \\
&= \sum_{m,n>0} \left[\frac{1}{8} J_{-m,-n}\tilde{J}_{-m,-n} + \frac{1}{16} J^{IJ}_{-m,-n}\tilde{J}^{IJ}_{-m,-n} + \frac{1}{384} J^{IJKL}_{-m,-n}\tilde{J}^{IJKL}_{-m,-n}\right] \ .
\end{aligned} \tag{B.5}
$$

where the second line (with the definition (2.7) for both the left and right moving variables) can be derived by using the Fiertz identity (G.11) and some of the trace formulas in (G.14). As we have gotten rid of the spinor indices we may now hope to evaluate the action of $\mathcal{T}$,

$$\begin{aligned}\mathcal{T}X\mathcal{T}^{-1} = X_{\bar{M}^V} \quad = \quad & \sum_{m,n>0}\left[\frac{1}{8}J_{-m,-n}\tilde{J}_{-m,-n} + \frac{1}{16}\sum_{IJ}\bar{\lambda}_{\{IJ\}}J^{IJ}_{-m,-n}\tilde{J}^{IJ}_{-m,-n} + \right. \\ & \left. \frac{1}{384}\sum_{IJKL}\bar{\lambda}_{\{IJKL\}}J^{IJKL}_{-m,-n}\tilde{J}^{IJKL}_{-m,-n}\right] \ . \end{aligned} \tag{B.6}$$

The last term on the right hand side with free summations over all the four indices is zero because of eq.(2.14) and the self-duality of $\tilde{J}^{IJKL}_{m,n}$ (eq.(2.9)). There are various ways to see that this is not correct. One can study the non-BPS boundary state in NSR formalism and explicitly check that the states that are created by this term are present. One can also argue in the current algebraic framework. If this term were zero then it would have implied that $\mathcal{T}$ annihilates some of the states in $\Pi^{(e)}$. On the other hand since $\left(\bar{M}^V\right)^2 = \mathbb{1}_8$, $\mathcal{T}^2$ should be expected to be identity. This would, in turn, mean that $\mathcal{T}$ is well-defined only in a subspace of $\Pi^{(e)}$. But action of $\mathcal{T}$ in eqs.(A.8) guarantees that it should be well-defined on all over $\Pi^{(e)}$ [14]. The origin of this problem is simply the "wrong" way of writing X in eq.(B.5). The last term on the right hand side of eq.(B.5) should be written in such a way that the summation over the tensor indices is restricted to the set $\mathcal{K}$ for reason that has been explained in sec.2. This immediately leads to,

$$|\text{even}\rangle = |\bar{M}^V, \text{even}\rangle \ . \tag{B.7}$$

Plugging in the result (B.3) into the third equation of (B.1) one gets,

$$|\text{odd}\rangle \quad = \quad -\mathcal{T}\left[\frac{\sinh\left(\sqrt{X}\right)}{\sqrt{X}}\left(\sum_{n>0}S^a_{-n}\tilde{S}^a_{-n}|\dot{a}\rangle\otimes|\widetilde{\dot{a}}\rangle\right)\right] \tag{B.8}$$

Using the Fiertz identity (G.13) one can further manipulate [24],

$$\sum_{n>0}S^a_{-n}\tilde{S}^a_{-n}|\dot{a}\rangle\otimes|\widetilde{\dot{a}}\rangle = Y^{\dot{a}\dot{b}}|\dot{a}\rangle\otimes|\widetilde{\dot{b}}\rangle \tag{B.9}$$

where,

$$Y^{\dot{a}\dot{b}} \quad = \quad \sum_{n=1}^{\infty}\left[\frac{1}{8}\sum_{I}\gamma^I_{a\dot{a}}\gamma^I_{b\dot{b}}S^a_{-n}\tilde{S}^b_{-n} + \frac{1}{48}\sum_{IJK}\gamma^{IJK}_{a\dot{a}}\gamma^{IJK}_{b\dot{b}}S^a_{-n}\tilde{S}^b_{-n}\right] \ , \tag{B.10}$$

Using this form it is now straightforward to find the action of $\mathcal{T}$ in eq.(B.8). One gets,

$$|\text{odd}\rangle \quad = \quad |\bar{M}^V, \text{odd}\rangle \ , \tag{B.11}$$

C Open-Closed Duality

Here we shall verify the open-closed world-sheet duality by using the boundary state (2.3, 2.4) and the open string spectrum obtained in the BCFT discussion (see subsecs.3.1 and 3.2.1). We shall show that the duality is satisfied only for the desired value of the normalization constant $\bar{\mathcal{N}}_p$ in eq.(2.3).

The form of the fermionic part of the boundary state as seen in eqs.(2.4, 2.5, 2.6) looks quite complicated. Also the commutation relations among the J oscillators in eq.(2.7) are not simple enough to result in an easy computation of the cylinder diagram in closed string channel. However, using the bosonization and refermionization method of sec.2 this computation can be enormously simplified. This is because of the simplified form of $|e\rangle\rangle$ in eq.(2.18) and the fact that the light-cone hamiltonian takes the same form both in terms of the oscillators of $\tilde{S}^a(\bar{z})$ and $\bar{S}^a(\bar{z})$.

$$\begin{aligned} H_c &= \frac{p^2}{2} + \sum_{n>0,I}\left(\alpha^I_{-n}\alpha^I_n + \tilde{\alpha}^I_{-n}\tilde{\alpha}^I_n\right) + \sum_{n>0,a} n\left(S^a_{-n}S^a_n + \tilde{S}^a_{-n}\tilde{S}^a_n\right) , \\ &= \frac{p^2}{2} + \sum_{n>0,I}\left(\alpha^I_{-n}\alpha^I_n + \tilde{\alpha}^I_{-n}\tilde{\alpha}^I_n\right) + \sum_{n>0,a} n\left(S^a_{-n}S^a_n + \bar{S}^a_{-n}\bar{S}^a_n\right) . \end{aligned} \tag{C.1}$$

The closed string channel expression for the partition function of open strings ending on the non-BPS D-brane is given by,

$$\begin{aligned} Z &= \int_0^\infty dl Z(l) , \\ Z(l) &= \langle \mathrm{B}\cancel{\mathrm{P}}\mathrm{S}, p| e^{-2\pi l H_c} | \mathrm{B}\cancel{\mathrm{P}}\mathrm{S}, p\rangle , \end{aligned} \tag{C.2}$$

Standard computations give the following results,

$$Z(l) = Z_0(l).Z_B(l).Z_F(l) , \tag{C.3}$$

where $Z_0(l)$, $Z_B(l)$ and $Z_F(l)$ are the contributions coming from the bosonic zero-modes, bosonic oscillators and fermionic oscillators respectively. Using the inner product $\langle k'_\perp | k_\perp\rangle = \delta^{(9-p)}(k'_\perp - k_\perp)$ and defining $q = e^{-2\pi l}$ we get,

$$Z_0(l) = \left(\bar{\mathcal{N}}_p\right)^2 l^{-(9-p)/2} , \quad Z_B(l) = \frac{q^{2/3}}{f_1(q)^8} , \quad Z_F(l) = \frac{1}{2} q^{-2/3} f_2(q)^8 . \tag{C.4}$$

The standard functions $f_i(q)$ that are relevant for our computations are given by,

$$f_1(q) = q^{1/12}\prod_{n=1}^\infty (1-q^{2n}) , \quad f_2(q) = \sqrt{2}q^{1/12}\prod_{n=1}^\infty (1+q^{2n}) , \quad f_4(q) = q^{-1/24}\prod_{n=1}^\infty (1-q^{2n-1}) ,$$

(C.5)

Defining $t = 1/2l$, $\tilde{q} = e^{-\pi t}$ and using the transformation properties,

$$f_1(e^{-\pi/t}) = \sqrt{t} f_1(e^{-\pi t}) \ , \quad f_2(e^{-\pi/t}) = f_4(e^{-\pi t}) \ , \tag{C.6}$$

one gets,

$$Z = \left(\bar{\mathcal{N}}_p\right)^2 2^{(5-p)/2} \int_0^\infty dt\ t^{-(3+p)/2} \left(\frac{f_4(\tilde{q})}{f_1(\tilde{q})}\right)^8 \ . \tag{C.7}$$

This has to be compared with the open string channel expression,

$$Z = \int_0^\infty \frac{dt}{2t} \left[\mathrm{Tr}_{NS}\left(e^{-2\pi t H_o}(-1)^F\right) + \mathrm{Tr}_R\left(e^{-2\pi t H_o}(-1)^F\right)\right] \ , \tag{C.8}$$

where F is the total space-time fermion number of a particular state. The Hamiltonian is given in the NS and R sectors as,

$$H_o = \begin{cases} (\vec{p})^2 + \sum_{n>0,I} \alpha^I_{-n}\alpha^I_n + \sum_{r>0,a} r S^a_{-r} S^a_r - \frac{1}{2} \ , & \text{(NS)} \ , \\ (\vec{p})^2 + \sum_{n>0,I} \alpha^I_{-n}\alpha^I_n + \sum_{n>0,a} n S^a_{-n} S^a_n \ , & \text{(R)} \ , \end{cases} \tag{C.9}$$

where $\vec{p}$ is the open string momentum along the world-volume of the D-brane. Since the R sector spectrum contains equal number of bosons and fermions at every level we have,

$$\mathrm{Tr}_R\left(e^{-2tH_o}(-1)^F\right) = 0 \ . \tag{C.10}$$

The trace in the NS sector gives the following result,

$$Z = \frac{V_{p+1}}{(2\pi)^{p+1}} 2^{-(3+p)/2} \int_0^\infty dt\ t^{-(3+p)/2} \left(\frac{f_4(\tilde{q})}{f_1(\tilde{q})}\right)^8 \ , \tag{C.11}$$

where we have used $\langle\vec{p}|\vec{p}\rangle = \delta^{(p+1)}(0) = V_{p+1}/(2\pi)^{p+1}$, V_{p+1} being the (infinite) volume of the $(p+1)$ dimensional world-volume. The two results (C.3) and (C.11) match only for,

$$\left(\bar{\mathcal{N}}_p\right)^2 = \frac{V_{p+1}}{16(2\pi)^{p+1}} \ . \tag{C.12}$$

We shall now argue that the above equation implies Sen's result [5, 9] which relates $\bar{\mathcal{T}}_p$, the tension of a non-BPS D-brane in type IIB and $\mathcal{T}_p$, the tension of the corresponding BPS D-brane in type IIA,

$$\bar{\mathcal{T}}_p = \sqrt{2}\ \mathcal{T}_p \ . \tag{C.13}$$

Boundary state for the BPS D-brane in type IIA which has the same configuration as the non-BPS D-brane (2.3) in type IIB is given by,

$$\begin{aligned} |\mathrm{BPS},\eta\rangle_{(IIA)} &= \mathcal{N}_p \int d^{(9-p)}k_\perp \exp\left(\sum_{n>0}\frac{1}{n}\alpha^I_{-n}\bar{M}^V_{IJ}\tilde{\alpha}^J_{-n} - i\eta\sum_{n>0}S^a_{-n}\bar{M}^S_{a\dot{b}}\tilde{S}^{\dot{b}}_{-n}\right) \\ & \left[\bar{M}^V_{IJ}|I\rangle\otimes\widetilde{|J\rangle} - i\eta\bar{M}^C_{\dot{a}b}|\dot{a}\rangle\otimes\widetilde{|b\rangle}\right]\otimes|k_\perp\rangle \ . \end{aligned} \tag{C.14}$$

A computation similar to the one performed above shows that the world-sheet duality is satisfied for the following value of the normalization constant.

$$\mathcal{N}_p^2 = \frac{V_{p+1}}{32(2\pi)^2} \ . \tag{C.15}$$

Now noticing that $\bar{\mathcal{N}}_p$ and $\mathcal{N}_p$ are the strengths of the massless NSNS state

$$\int d^{(9-p)}k_\perp \bar{M}^V_{IJ}|I\rangle\otimes\widetilde{|J\rangle}\otimes|k_\perp\rangle \ , \tag{C.16}$$

in the boundary states $|\,\mathrm{B}\not{\mathrm{P}}\mathrm{S}\rangle$ and $|\mathrm{BPS},\eta\rangle_{(IIA)}$ respectively, eqs.(C.12) and (C.15) imply (C.13).

D Open String Boundary Condition

Here we shall derive the open string boundary condition (3.9) from a certain quadratic gluing condition satisfied by the covariant expression (2.4). Although the set of oscillators J_{mn}, J^{IJ}_{mn} and J^{IJKL}_{mn} do not have simple commutation relations among themselves and finding such a gluing condition seems to be difficult, one can actually simplify this task by using the bosonization and refermionization trick of sec.2. Since $|\mathrm{BPS},-1,\eta\rangle_{\bar{S}}$ takes the form of the fermionic part of a BPS D-instanton boundary state with the replacement indicated below eq.(2.18), it satisfies the gluing condition (2.2) with the same replacement and with $M^S_{ab}=\delta_{ab}$. Using this gluing condition and eq.(2.18) one readily finds [24],

$$S^a_m S^b_n|e\rangle\rangle = \bar{S}^b_{-n}\bar{S}^a_{-m}|e\rangle\rangle \ , \quad \forall m,n\in \mathbb{Z} \ . \tag{D.1}$$

This can be re-written in terms of the local fields on the cylinder in the closed string channel as,

$$\left[S^a(0,\sigma)S^b(0,\sigma') - \bar{S}^b(0,\sigma')\bar{S}^a(0,\sigma)\right]|e\rangle\rangle = 0 \ , \tag{D.2}$$

where the first argument is the value of the world-sheet time τ in closed string channel. Using the standard method one can go to the open string channel to get the following open string boundary condition on the upper half plane,

$$S^a(z)S^b(w) + \bar{S}^b(\bar{w})\bar{S}^a(\bar{z}) = 0 \ , \quad \text{at } z = \bar{z}, \ w = \bar{w} \ . \tag{D.3}$$

Notice that for $z \neq w$ one can write,

$$S^a(z)S^b(w) = \bar{S}^a(\bar{z})\bar{S}^b(\bar{w}) \ , \quad \text{at } z = \bar{z}, \ w = \bar{w} \ . \tag{D.4}$$

One can also argue that this equation is correct even when $z = w$ if we take the operators to be normal ordered. We can now rewrite the right hand side in terms of $\tilde{S}^a(\bar{z})$:

$$\begin{aligned} \bar{S}^a(\bar{z})\bar{S}^b(\bar{w}) &= \frac{1}{8}\delta_{ab}\left(\bar{S}^c(\bar{z})\bar{S}^c(\bar{w})\right) + \frac{1}{16}\sum_{IJ}\gamma^{IJ}_{ab}\left(\bar{S}^c(\bar{z})\gamma^{IJ}_{cd}\bar{S}^d(\bar{w})\right) + \\ &\quad \frac{1}{384}\sum_{IJKL}\gamma^{IJKL}_{ab}\left(\bar{S}^c(\bar{z})\gamma^{IJKL}_{cd}\bar{S}^d(\bar{w})\right) \ , \\ &= \frac{1}{8}\delta_{ab}\left(\tilde{S}^c(\bar{z})\tilde{S}^c(\bar{w})\right) + \frac{1}{16}\sum_{IJ}\bar{\lambda}_{\{IJ\}}\gamma^{IJ}_{ab}\left(\tilde{S}^c(\bar{z})\gamma^{IJ}_{cd}\tilde{S}^d(\bar{w})\right) + \\ &\quad \frac{2}{384}\sum_{\{IJKL\}\in\mathcal{K}}\bar{\lambda}_{\{IJKL\}}\gamma^{IJKL}_{ab}\left(\tilde{S}^c(\bar{z})\gamma^{IJKL}_{cd}\tilde{S}^d(\bar{w})\right) \ . \end{aligned} \tag{D.5}$$

We have used the Fiertz identity (G.11) in the first step where all the tensor indices are summed over freely. In the second step we have performed the bosonization and refermionization trick of sec.2 in the reverse direction. Finally, using the result (D.5) in eq.(D.4) one arrives at the condition (3.9).

E One-Point Functions

Here we shall verify the relation (3.22) with $C = 1$ for the states $|\Phi^{(IJK),(MNP)}_{m,n}\rangle$, $|\Phi^{IJ}_{m,n,p,q}\rangle$ $|\Phi^{(MN),(M'N'),IJ}_{m,n,p,q}\rangle$ and $|\Phi^{(MNPQ),(M'N'P'Q'),IJ}_{m,n,p,q}\rangle$ as defined in eqs.(3.23). The corresponding vertex operators are given by,

$$\begin{aligned} \Phi^{(IJK),(MNP)}_{m,n}(\zeta,\bar{\zeta}) &= -\gamma^{IJK}_{a\dot{a}}\gamma^{MNP}_{b\dot{b}}\Phi^{ab\dot{a}\dot{b}}_{m,n}(\zeta,\bar{\zeta}) \ , \\ \Phi^{IJ}_{m,n,p,q}(\zeta,\bar{\zeta}) &= \Phi^{aabbIJ}_{m,n,p,q}(\zeta,\bar{\zeta}) \ , \\ \Phi^{(MN),(M'N'),IJ}_{m,n,p,q}(\zeta,\bar{\zeta}) &= \gamma^{MN}_{ab}\gamma^{M'N'}_{cd}\Phi^{abcdIJ}_{m,n,p,q}(\zeta,\bar{\zeta}) \ , \end{aligned}$$

$$\Phi^{(MNPQ),(M'N'P'Q'),IJ}_{m,n,p,q}(\zeta,\bar\zeta) \;=\; \gamma^{MNPQ}_{ab}\gamma^{M'N'P'Q'}_{cd}\,\Phi^{abcdIJ}_{m,n,p,q}(\zeta,\bar\zeta)\ , \tag{E.1}$$

respectively. The operator $\Phi^{ab\dot a\dot b}_{m,n}(\zeta,\bar\zeta)$ is defined in eq.(3.25) and,

$$\begin{aligned}\Phi^{abcdIJ}_{m,n,p,q}(\zeta,\bar\zeta) \;=\; & \oint_\zeta \frac{d\zeta_1 d\zeta_2}{(2\pi i)^2}\oint_{\bar\zeta}\frac{d\bar\zeta_3 d\bar\zeta_4}{(2\pi i)^2}(\zeta_1-\zeta)^{-m-1/2}(\zeta_2-\zeta)^{-n-1/2} \\ & (\bar\zeta_3-\bar\zeta)^{-p-1/2}(\bar\zeta_4-\bar\zeta)^{-q-1/2}S^a(\zeta_1)S^b(\zeta_2)\tilde S^c(\bar\zeta_3)\tilde S^d(\bar\zeta_4)\psi^I(\zeta)\tilde\psi^J(\bar\zeta)\ .\end{aligned} \tag{E.2}$$

The non-trivial inner products can be computed to be,

$$\begin{aligned}
\langle\Phi^{(IJK),(MNP)}_{m,n}|\Phi^{(I'J'K'),(M'N'P')}_{m',n'}\rangle \;&=\; 64\delta_{m,m'}\ \delta_{n,n'}\ \Delta_{(IJK),(I'J'K')}\Delta_{(MNP),(M'N'P')}\ , \\
\langle\Phi^{IJ}_{m,n,p,q}|\Phi^{I'J'}_{m',n',p',q'}\rangle \;&=\; 64(\delta_{m,m'}\ \delta_{n,n'}-\delta_{m,n'}\ \delta_{n,m'}) \\
&\quad (\delta_{p,p'}\ \delta_{q,q'}-\delta_{p,q'}\ \delta_{q,p'})\delta_{I,I'}\ \delta_{J,J'}\ , \\
\langle\Phi^{(KL),(MN),IJ}_{m,n,p,q}|\Phi^{(K'L'),(M'N'),I'J'}_{m',n',p',q'}\rangle \;&=\; 64(\delta_{m,m'}\ \delta_{n,n'}+\delta_{m,n'}\ \delta_{n,m'}) \\
&\quad (\delta_{p,p'}\ \delta_{q,q'}+\delta_{p,q'}\ \delta_{q,p'})\delta_{I,I'}\ \delta_{J,J'} \\
&\quad \Delta_{(KL),(K'L')}\Delta_{(MN),(M'N')}\ , \\
\langle\Phi^{(KLMN),(PQRS),IJ}_{m,n,p,q}|\Phi^{(K'L'M'N'),(P'Q'R'S'),I'J'}_{m',n',p',q'}\rangle \;&=\; 64(\delta_{m,m'}\ \delta_{n,n'}-\delta_{m,n'}\ \delta_{n,m'}) \\
&\quad (\delta_{p,p'}\ \delta_{q,q'}-\delta_{p,q'}\ \delta_{q,p'})\delta_{I,I'}\ \delta_{J,J'} \\
&\quad \Delta_{(KLMN),(K'L'M'N')}\Delta_{(PQRS),(P'Q'R'S')}\ , \\
&\quad \text{for } \{K,L,M,N\},\{K',L',M',N'\}, \\
&\quad \{P,Q,R,S\},\{P',Q',R',S'\}\in\mathcal{K}\ .
\end{aligned} \tag{E.3}$$

where we have introduced,

$$\Delta_{(I_1\cdots I_n),(I'_1\cdots I'_n)} \equiv \sum_{\mathcal P}\mathrm{sign}\mathcal{P}\ \delta_{\{I_1\cdots I_n\},\mathcal{P}\{I'_1\cdots I'_n\}}\ . \tag{E.4}$$

The summation goes over $n!$ terms. $\{I_1,\cdots I_n\}$ is an ordered set while $\mathcal{P}\{I_1,\cdots I_n\}$ is another ordered set obtained by applying the permutation $\mathcal P$ on $\{I_1,\cdots I_n\}$. sign$\mathcal P$ is 1 if $\mathcal P$ is even and (-1) otherwise. $\delta_{\{I_1\cdots I_n\},\{I'_1\cdots I'_n\}}$ is the n-dimensional Kronecker delta function which is 1 only when the two ordered sets $\{I_1\cdots I_n\}$ and $\{I'_1\cdots I'_n\}$ are equal and zero otherwise. Using the above inner products and eq.(2.4) one gets the following

results,

$$\langle\langle e|\Phi^{(IJK),(MNP)}_{m,n}\rangle \;=\; 8\delta_{m,n}\;\bar{\lambda}_{\{IJK\}}\Delta_{(IJK),(MNP)}\;, \tag{E.5}$$

$$\langle\langle e|\Phi^{IJ}_{m,n,p,q}\rangle \;=\; 8(\delta_{m,p}\;\delta_{n,q}-\delta_{m,q}\;\delta_{n,p})\bar{\lambda}_I\delta_{IJ}\;, \tag{E.6}$$

$$\langle\langle e|\Phi^{(MN),(PQ),IJ}_{m,n,p,q}\rangle \;=\; 8(\delta_{m,p}\;\delta_{n,q}+\delta_{m,q}\;\delta_{n,p})\bar{\lambda}_I\bar{\lambda}_{\{MN\}}\delta_{IJ}\Delta_{(MN),(PQ)}\;, \tag{E.7}$$

$$\begin{aligned}\langle\langle e|\Phi^{(KLMN),(PQRS),IJ}_{m,n,p,q}\rangle \;&=\; 8(\delta_{m,p}\;\delta_{n,q}-\delta_{m,q}\;\delta_{n,p})\bar{\lambda}_I\bar{\lambda}_{\{KLMN\}}\delta_{IJ}\Delta_{(KLMN),(PQRS)}\;,\\ &\quad\text{for } \{K,L,M,N\},\{P,Q,R,S\}\in\mathcal{K}\;.\end{aligned} \tag{E.8}$$

By going through an analysis similar to the one done in subsec. 3.2.2 one can write the one-point functions of the vertices in (E.1) in terms of certain correlation functions on the full plane.

$$\begin{aligned}\langle\Phi^{(IJK),(MNP)}_{m,n}(0,0)\rangle_D \;&=\; \bar{\lambda}_{\{MNP\}}\oint_i\frac{dz}{2\pi i}\oint_{-i}\frac{dw}{2\pi i}\mathcal{J}_{m,n}(z,w)\\ &\quad\gamma^{IJK}_{a\dot{a}}\gamma^{MNP}_{b\dot{b}}\langle\mathcal{S}^a(z)\mathcal{S}^b(w)\mathcal{S}^{\dot{a}}(i)\mathcal{S}^{\dot{b}}(-i)\rangle\;,\end{aligned} \tag{E.9}$$

$$\begin{aligned}\langle\Phi^{IJ}_{m,n,p,q}(0,0)\rangle_D \;&=\; \bar{\lambda}_J\oint_i\frac{dz_1dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)\\ &\quad\langle\mathcal{S}^a(z_1)\mathcal{S}^a(w_1)\mathcal{S}^c(z_2)\mathcal{S}^c(w_2)\Psi^I(i)\Psi^J(-i)\rangle\;,\end{aligned} \tag{E.10}$$

$$\begin{aligned}\langle\Phi^{(MN),(PQ),IJ}_{m,n,p,q}(0,0)\rangle_D \;&=\; \bar{\lambda}_J\bar{\lambda}_{\{PQ\}}\oint_i\frac{dz_1dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)\\ &\quad\gamma^{MN}_{ab}\gamma^{PQ}_{cd}\langle\mathcal{S}^a(z_1)\mathcal{S}^b(w_1)\mathcal{S}^c(z_2)\mathcal{S}^d(w_2)\Psi^I(i)\Psi^J(-i)\rangle\;,\end{aligned} \tag{E.11}$$

$$\begin{aligned}\langle\Phi^{(KLMN),(PQRS),IJ}_{m,n,p,q}(0,0)\rangle_D \;&=\; \bar{\lambda}_J\bar{\lambda}_{\{PQRS\}}\oint_i\frac{dz_1dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)\\ &\quad\gamma^{KLMN}_{ab}\gamma^{PQRS}_{cd}\langle\mathcal{S}^a(z_1)\mathcal{S}^b(w_1)\mathcal{S}^c(z_2)\mathcal{S}^d(w_2)\Psi^I(i)\Psi^J(-i)\rangle\;,\end{aligned} \tag{E.12}$$

where $\mathcal{J}_{m,n}(z,w)$ is given in eq.(3.34) and,

$$\begin{aligned}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2) \;&=\; 8(z_1-i)^{-m-1/2}(z_1+i)^{m-1/2}(w_1-i)^{-n-1/2}(w_1+i)^{n-1/2}\\ &\quad(z_2+i)^{-p-1/2}(z_2-i)^{p-1/2}(w_2+i)^{-q-1/2}(w_2-i)^{q-1/2}\;.\end{aligned}$$

(E.13)

The correlation functions appearing in the above integrands have been computed in appendix F in terms of certain functions. The full set of results of the required integrals involving these functions have been listed in appendix H. To compute the right hand sides of eqs. (E.9), (E.10), (E.11) and (E.12) one needs to use eqs. (H.16), (H.17), (H.18, H.19, H.20) and (H.21, H.22, H.23) respectively. The final results show equality of the one-point functions (E.9), (E.10), (E.11), (E.12) with the boundary state results (E.5), (E.6), (E.7), (E.8) respectively.

F Correlation Functions on the Plane

Certain 4 and 6-point correlation functions on the full plane are needed to compute the closed string one-point functions (3.35), (E.9), (E.10), (E.11) and (E.12). The index structures and various symmetry properties allow us to write down these correlation functions in the following forms,

$$\gamma^I_{a\dot{a}}\gamma^J_{b\dot{b}}\langle\mathcal{S}^a(z)\mathcal{S}^b(w)\mathcal{S}^{\dot{a}}(u)\mathcal{S}^{\dot{b}}(v)\rangle = \delta_{IJ}\tilde{F}^{(1,1)}(z,w,u,v) \ , \tag{F.1}$$

$$\gamma^{IJK}_{a\dot{a}}\gamma^{MNP}_{b\dot{b}}\langle\mathcal{S}^a(z)\mathcal{S}^b(w)\mathcal{S}^{\dot{a}}(u)\mathcal{S}^{\dot{b}}(v)\rangle = \Delta_{(IJK),(MNP)}\tilde{F}^{(3,3)}(z,w,u,v) \ , \tag{F.2}$$

$$\langle\mathcal{S}^a(z_1)\mathcal{S}^a(w_1)\mathcal{S}^c(z_2)\mathcal{S}^c(w_2)\Psi^I(u)\Psi^J(v)\rangle = \delta_{IJ}F^{(1,1)}(z_1,w_1,z_2,w_2,u,v) \ , \tag{F.3}$$

$$\begin{aligned} &\gamma^{MN}_{ab}\gamma^{PQ}_{cd}\langle\mathcal{S}^a(z_1)\mathcal{S}^b(w_1)\mathcal{S}^c(z_2)\mathcal{S}^d(w_2)\Psi^I(u)\Psi^J(v)\rangle \\ = \ &\delta_{IJ}\Delta_{(MN),(PQ)}F_1^{(3,3)}(z_1,w_1,z_2,w_2,u,v)+ \\ &\Big(\delta_{QJ}\Delta_{(MN),(IP)}+\delta_{PJ}\Delta_{(MN),(QI)}\Big)\,F_2^{(3,3)}(z_1,w_1,z_2,w_2,u,v)- \\ &\Big(\delta_{QI}\Delta_{(MN),(JP)}+\delta_{PI}\Delta_{(MN),(QJ)}\Big)\,F_2^{(3,3)}(z_1,w_1,z_2,w_2,v,u) \ , \end{aligned} \tag{F.4}$$

$$\begin{aligned} &\gamma^{KLMN}_{ab}\gamma^{PQRS}_{cd}\langle\mathcal{S}^a(z_1)\mathcal{S}^b(w_1)\mathcal{S}^c(z_2)\mathcal{S}^d(w_2)\Psi^I(u)\Psi^J(v)\rangle \\ = \ &\delta_{IJ}\left(\Delta_{(KLMN),(PQRS)}+\epsilon^{KLMNPQRS}\right)F_1^{(5,5)}(z_1,w_1,z_2,w_2,u,v)+ \\ &D_{(KLMN),(PQRS),IJ}F_2^{(5,5)}(z_1,w_1,z_2,w_2,u,v)-D_{(KLMN),(PQRS),JI}F_2^{(5,5)}(z_1,w_1,z_2,w_2,v,u) \ , \end{aligned} \tag{F.5}$$

where the notation $\Delta_{(IJ\cdots)(KL\cdots)}$ has been introduced in eq.(E.4) and

$$\begin{aligned}
D_{(KLMN),(PQRS),IJ} &= \delta_{KI}\Big(\delta_{PJ}\Delta_{(LMN),(QRS)} - \delta_{QJ}\Delta_{(LMN),(PRS)} \\
&\quad -\delta_{RJ}\Delta_{(LMN),(QPS)} - \delta_{SJ}\Delta_{(LMN),(QRP)}\Big) \\
&\quad -\delta_{LI}\Big(\delta_{PJ}\Delta_{(KMN),(QRS)} - \delta_{QJ}\Delta_{(KMN),(PRS)} \\
&\quad -\delta_{RJ}\Delta_{(KMN),(QPS)} - \delta_{SJ}\Delta_{(KMN),(QRP)}\Big) \\
&\quad -\delta_{MI}\Big(\delta_{PJ}\Delta_{(LKN),(QRS)} - \delta_{QJ}\Delta_{(LKN),(PRS)} \\
&\quad -\delta_{RJ}\Delta_{(LKN),(QPS)} - \delta_{SJ}\Delta_{(LKN),(QRP)}\Big) \\
&\quad -\delta_{NI}\Big(\delta_{PJ}\Delta_{(LMK),(QRS)} - \delta_{QJ}\Delta_{(LMK),(PRS)} \\
&\quad -\delta_{RJ}\Delta_{(LMK),(QPS)} - \delta_{SJ}\Delta_{(LMK),(QRP)}\Big) \ . \qquad \text{(F.6)}
\end{aligned}$$

The various functions have the following symmetry properties,

$$\tilde{F}^{(1,1)}(z,w,u,v) = \tilde{F}^{(1,1)}(w,z,v,u) \ , \qquad \text{(F.7)}$$

$$\tilde{F}^{(3,3)}(z,w,u,v) = \tilde{F}^{(3,3)}(w,z,v,u) \ , \qquad \text{(F.8)}$$

$$\begin{aligned}
F^{(1,1)}(z_1,w_1,z_2,w_2,u,v) &= -F^{(1,1)}(w_1,z_1,z_2,w_2,u,v) \ , \\
&= -F^{(1,1)}(z_1,w_1,w_2,z_2,u,v) \ , \\
&= -F^{(1,1)}(z_1,w_1,z_2,w_2,v,u) \ , \\
&= F^{(1,1)}(z_2,w_2,z_1,w_1,u,v) \ , \qquad \text{(F.9)}
\end{aligned}$$

$$\begin{aligned}
F_1^{(3,3)}(z_1,w_1,z_2,w_2,u,v) &= F_1^{(3,3)}(w_1,z_1,z_2,w_2,u,v) \ , \\
&= F_1^{(3,3)}(z_1,w_1,w_2,z_2,u,v) \ , \\
&= -F_1^{(3,3)}(z_1,w_1,z_2,w_2,v,u) \ , \\
&= F_1^{(3,3)}(z_2,w_2,z_1,w_1,u,v) \ , \qquad \text{(F.10)}
\end{aligned}$$

$$\begin{aligned}
F_2^{(3,3)}(z_1,w_1,z_2,w_2,u,v) &= F_2^{(3,3)}(w_1,z_1,z_2,w_2,u,v) \ , \\
&= F_2^{(3,3)}(z_1,w_1,w_2,z_2,u,v) \ , \qquad \text{(F.11)}
\end{aligned}$$

$$\begin{aligned}
F_1^{(5,5)}(z_1,w_1,z_2,w_2,u,v) &= -F_1^{(5,5)}(w_1,z_1,z_2,w_2,u,v) \ , \\
&= -F_1^{(5,5)}(z_1,w_1,w_2,z_2,u,v) \ , \\
&= -F_1^{(5,5)}(z_1,w_1,z_2,w_2,v,u) \ , \\
&= F_1^{(5,5)}(z_2,w_2,z_1,w_1,u,v) \ , \qquad \text{(F.12)}
\end{aligned}$$

$$
\begin{aligned}
F_2^{(5,5)}(z_1,w_1,z_2,w_2,u,v) &= -F_2^{(5,5)}(w_1,z_1,z_2,w_2,u,v) \ , \\
&= -F_2^{(5,5)}(z_1,w_1,w_2,z_2,u,v) \ , \\
&= -F_2^{(5,5)}(z_2,w_2,z_1,w_1,v,u) \ . \qquad \text{(F.13)}
\end{aligned}
$$

To compute the above functions we bosonize the holomorphic fermions $\mathcal{S}^a(z)$, $\mathcal{S}^{\dot{a}}(z)$ and $\Psi^I(z)$ and use Mathematica. This method requires a full set of consistent formulas for the basis vectors for the various weight lattices, cocycle factors and the corresponding gamma matrix representation. We take the sets of basis vectors e_i, δ_i and $\bar{\delta}_i$ $(i = 1, \cdots, 4)$ for the vector, spinor and conjugate spinor weight lattices respectively to be as given in [14]. For cocycle factors and the corresponding gamma matrices we follow the convention of [30]. The explicit formulas can be found in appendix G. We give the final results below. The functions involved in the four point correlators are computed to be,

$$
\tilde{F}^{(1,1)}(z,w,u,v) = -\frac{24i}{\sqrt{(z-u)(z-v)(w-u)(w-v)}} + \frac{8i\sqrt{(z-v)(w-u)}}{(u-v)(z-w)\sqrt{(z-u)(w-v)}} \ , \qquad \text{(F.14)}
$$

$$
\tilde{F}^{(3,3)}(z,w,u,v) = \frac{8i\sqrt{(z-u)(w-v)}}{(u-v)(z-w)\sqrt{(z-v)(w-u)}} \ , \qquad \text{(F.15)}
$$

The functions involved in the six-point correlators are given by,

$$
\begin{aligned}
F^{(1,1)}(z_1,w_1,z_2,w_2,u,v) &= F_1(z_1,w_1,z_2,w_2,u,v) - F_1(w_1,z_1,z_2,w_2,u,v) - \\
&\quad F_1(z_1,w_1,w_2,z_2,u,v) - F_1(z_1,w_1,z_2,w_2,v,u) + \\
&\quad F_2(z_1,w_1,z_2,w_2,u,v) - F_2(w_1,z_1,z_2,w_2,u,v) - \\
&\quad F_2(z_1,w_1,w_2,z_2,u,v) - F_2(z_1,w_1,z_2,w_2,v,u) \ , \qquad \text{(F.16)}
\end{aligned}
$$

$$
\begin{aligned}
F_1^{(3,3)}(z_1,w_1,z_2,w_2,u,v) &= \frac{1}{3}F_1(z_1,w_2,z_2,w_1,u,v) + \frac{1}{3}F_1(w_1,w_2,z_2,z_1,u,v) + \\
&\quad \frac{1}{3}F_1(z_1,z_2,w_2,w_1,u,v) - \frac{1}{3}F_1(z_1,w_2,z_2,w_1,v,u) \ , \qquad \text{(F.17)}
\end{aligned}
$$

$$
\begin{aligned}
F_2^{(3,3)}(z_1,w_1,z_2,w_2,u,v) &= F_3(z_1,w_1,z_2,w_2,u,v) + F_3(w_1,z_1,z_2,w_2,u,v) + \\
&\quad F_3(z_1,w_1,w_2,z_2,u,v) + F_3(w_1,z_1,w_2,z_2,u,v) \ , \qquad \text{(F.18)}
\end{aligned}
$$

$$
\begin{aligned}
F_1^{(5,5)}(z_1,w_1,z_2,w_2,u,v) &= -\frac{1}{3}F_1(z_1,w_2,z_2,w_1,u,v) + \frac{1}{3}F_1(w_1,w_2,z_2,z_1,u,v) \\
&\quad + \frac{1}{3}F_1(z_1,w_2,z_2,w_1,v,u) + \frac{1}{3}F_1(z_1,z_2,w_2,w_1,u,v) \ , \qquad \text{(F.19)}
\end{aligned}
$$

$$
F_2^{(5,5)}(z_1,w_1,z_2,w_2,u,v) = -F_2(z_1,w_2,z_2,w_1,u,v) - F_2(z_1,w_2,z_2,w_1,v,u) \ , \qquad \text{(F.20)}
$$

where

$$F_1(z_1,w_1,z_2,w_2,u,v) = \frac{12\sqrt{(z_1-u)(z_2-u)(w_1-v)(w_2-v)}}{(u-v)(z_1-w_1)(z_2-w_2)\sqrt{(z_1-v)(z_2-v)(w_1-u)(w_2-u)}} ,$$

$$F_2(z_1,w_1,z_2,w_2,u,v) = -\frac{4(z_1-w_2)(z_2-w_1)\sqrt{(z_1-v)(z_2-u)(w_1-u)(w_2-v)}}{(z_1-z_2)(w_1-w_2)(u-v)(z_1-w_1)(z_2-w_2)\sqrt{(z_1-u)(z_2-v)(w_1-v)(w_2-u)}} ,$$

$$F_3(z_1,w_1,z_2,w_2,u,v) = -\frac{4\sqrt{(w_1-v)(w_2-u)}}{(w_1-w_2)\sqrt{(z_1-u)(z_1-v)(w_1-u)(w_2-v)(z_2-u)(z_2-v)}} , \tag{F.21}$$

G Bosonization and Gamma Matrices

Here we present the basic formulas for the bosonization of the holomorphic fields $\mathcal{S}^a(z)$, $\mathcal{S}^{\dot{a}}(z)$, and $\Psi^I(z)$ on the full complex plane and the corresponding gamma matrix representation. We take the sets of four dimensional basis vectors e_i, δ_i and $\bar{\delta}_i$ $(i=1,\cdots,4)$ for the vector, spinor and conjugate spinor weight lattices respectively to be as given in [14],

$$\begin{array}{llll} e_1=(1,0,0,0), & e_2=(0,1,0,0), & e_3=(0,0,1,0), & e_4=(0,0,0,1) , \\ \delta_1=\frac{1}{2}(-1,1,-1,1), & \delta_2=\frac{1}{2}(-1,1,1,-1), & \delta_3=\frac{1}{2}(1,1,1,1), & \delta_4=\frac{1}{2}(1,1,-1,-1) , \\ \bar{\delta}_1=\frac{1}{2}(1,1,-1,1), & \bar{\delta}_2=\frac{1}{2}(1,1,1,-1), & \bar{\delta}_3=\frac{1}{2}(-1,1,1,1), & \bar{\delta}_4=\frac{1}{2}(-1,1,-1,-1) , \end{array} \tag{G.1}$$

The spinors corresponding to the above weight vectors are defined to be,

$$\begin{aligned} \lambda^{\pm e_j}(z) &= \frac{1}{\sqrt{2}}\left(\Psi^{\mu=2j-1}(z) \mp i\Psi^{\mu=2j}(z)\right) , \\ \chi^{\pm\delta_j}(z) &= \exp\left(-i\frac{\pi}{2}\delta_j.M_4\delta_j\right)\frac{1}{\sqrt{2}}\left(\mathcal{S}^{a=2j-1}(z) \mp i\mathcal{S}^{a=2j}(z)\right) , \\ \xi^{\pm\bar{\delta}_j}(z) &= \exp\left(-i\frac{\pi}{2}\bar{\delta}_j.M_4\bar{\delta}_j\right)\frac{1}{\sqrt{2}}\left(\mathcal{S}^{\dot{a}=2j-1}(z) \mp i\mathcal{S}^{\dot{a}=2j}(z)\right) , \end{aligned} \tag{G.2}$$

where,

$$M_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 \end{pmatrix} . \tag{G.3}$$

The extra phases in the last two equations in (G.2) will be explained later. The bosonization is given by,

$$\lambda^{\pm e_j}(z) = \exp\left(\pm i e_j.\phi\right)(z) C_{\pm e_j}(\hat{p}) ,$$

$$\begin{aligned} \chi^{\pm\delta_j}(z) &= \exp\left(\pm i\delta_j.\phi\right)(z) C_{\pm\delta_j}(\hat{p}) \ , \\ \xi^{\pm\bar{\delta}_j}(z) &= \exp\left(\pm i\bar{\delta}_j.\phi\right)(z) C_{\pm\bar{\delta}_j}(\hat{p}) \ , \end{aligned} \tag{G.4}$$

where $\phi(z)$ is a holomorphic scalar field normalized such that $\phi(z)\phi(w) \sim -\ln(z-w)$. Following [30] we take the cocycle factor $C_w(\hat{p})$ corresponding to the weight w to be,

$$C_w(\hat{p}) \ = \ \exp\left(i\pi w.M_4\hat{p}\right) \ , \tag{G.5}$$

where $\hat{p}$ is the momentum conjugate to the zero mode of $\phi(z)$. Defining the vector, spinor and conjugate spinor indices $\hat{I}$, A and $\dot{A} = 1, \cdots 8$ corresponding to the lattice basis such that,

$$\begin{aligned} &E_{\hat{I}} = \begin{cases} e_j & \text{for } \hat{I} = 2j-1 \\ -e_j & \text{for } \hat{I} = 2j \end{cases} \ , \quad \Delta_A = \begin{cases} \delta_j & \text{for } A = 2j-1 \\ -\delta_j & \text{for } A = 2j \end{cases} \ , \\ &\bar{\Delta}_{\dot{A}} = \begin{cases} \bar{\delta}_j & \text{for } \dot{A} = 2j-1 \\ -\bar{\delta}_j & \text{for } \dot{A} = 2j \end{cases} \ , \quad j = 1, \cdots, 4 \ , \end{aligned} \tag{G.6}$$

the corresponding gamma matrices [30][15] are given by,

$$\begin{aligned} (\gamma^{\hat{I}})^A{}_{\dot{A}} &= \sqrt{2}\exp\left(i\pi E_{\hat{I}}.M_4\bar{\Delta}_{\dot{A}}\right)\delta_{E_{\hat{I}}+\Delta_A, \bar{\Delta}_{\dot{A}}} \ , \\ (\bar{\gamma}^{\hat{I}})^{\dot{A}}{}_A &= \sqrt{2}\exp\left(i\pi E_{\hat{I}}.M_4\Delta_A\right)\delta_{E_{\hat{I}}+\bar{\Delta}_{\dot{A}}, \Delta_A} \ . \end{aligned} \tag{G.7}$$

The spinor indices are raised and lowered by the charge conjugation matrices,

$$\begin{aligned} C^{AB} &= \exp\left(-i\pi\Delta_A.M_4\Delta_A\right)\delta_{\Delta_A, -\Delta_B} \ , \\ C^{\dot{A}\dot{B}} &= i\exp\left(-i\pi\bar{\Delta}_{\dot{A}}.M_4\bar{\Delta}_{\dot{A}}\right)\delta_{\bar{\Delta}_{\dot{A}}, -\bar{\Delta}_{\dot{B}}} \ . \end{aligned} \tag{G.8}$$

The phases in the last two equations in (G.4) correspond to one choice such that the charge conjugation matrices are given by the identity matrix in the covariant basis, i.e. $\mathcal{S}^a(z)\mathcal{S}^b(z) \sim \frac{\delta^{ab}}{(z-w)}$, $\mathcal{S}^{\dot{a}}(z)\mathcal{S}^{\dot{b}}(z) \sim \frac{\delta^{\dot{a}\dot{b}}}{(z-w)}$. In this basis the 16-dimensional gamma matrices are hermitian: $\left(\Gamma^I\right)^\dagger = \Gamma^I$. Therefore,

$$\bar{\gamma}^I = \left(\gamma^I\right)^\dagger \ . \tag{G.9}$$

It turns out that any non-zero element of a γ^I is one of the following three phases: $e^{i\pi/4}$, $\pm ie^{-i\pi/4}$. This implies the following reality property,

$$\gamma^I_{a\dot{a}}\left(\gamma^J_{b\dot{b}}\right)^* = \text{ real} \ . \tag{G.10}$$

[15] The gamma matrix representation taken in [14] is consistent with this choice of the basis vectors on the weight lattices, but corresponds to different choice of cocycle factors which were not needed for the computations considered there.

Using the above reality and hermiticity properties one establishes that γ^{IJ} and $\bar{\gamma}^{IJ}$ are real antisymmetric while γ^{IJKL} and $\bar{\gamma}^{IJKL}$ are real symmetric.

Representation independent relations

Various Fiertz identities, which we have used very crucially in our computations, are given by,

$$\begin{aligned}
\lambda_1^a\lambda_2^b &= \frac{1}{8}(\lambda_1^c\lambda_2^c)\,\delta_{ab} + \frac{1}{16}(\lambda_1^c\gamma^{IJ}_{cd}\lambda_2^d)\,\gamma^{IJ}_{ab} + \frac{1}{384}(\lambda_1^c\gamma^{IJKL}_{cd}\lambda_2^d)\,\gamma^{IJKL}_{ab}, && \text{(G.11)}\\
\lambda_1^{\dot a}\lambda_2^{\dot b} &= \frac{1}{8}(\lambda_1^{\dot c}\lambda_2^{\dot c})\,\delta_{\dot a\dot b} + \frac{1}{16}(\lambda_1^{\dot c}\bar\gamma^{IJ}_{\dot c\dot d}\lambda_2^{\dot d})\,\bar\gamma^{IJ}_{\dot a\dot b} + \frac{1}{384}(\lambda_1^{\dot c}\bar\gamma^{IJKL}_{\dot c\dot d}\lambda_2^{\dot d})\,\bar\gamma^{IJKL}_{\dot a\dot b}, && \text{(G.12)}\\
\lambda_1^a\lambda_2^{\dot b} &= \frac{1}{8}(\lambda_1^c\gamma^I_{c\dot d}\lambda_2^{\dot d})\bar\gamma^I_{\dot b a} - \frac{1}{48}(\lambda_1^c\gamma^{IJK}_{c\dot d}\lambda_2^{\dot d})\bar\gamma^{IJK}_{\dot b a}. && \text{(G.13)}
\end{aligned}$$

These identities hold both in the real and the hermitian representation described above. These can be proved by using various representation independent relations summarized below:

$$\begin{aligned}
\mathrm{Tr}(\gamma^I\bar\gamma^J) &= 8\delta_{IJ}\ ,\\
\mathrm{Tr}(\gamma^I\bar\gamma^{JKL}) &= 0\ ,\\
\mathrm{Tr}(\gamma^{IJK}\bar\gamma^{MNP}) &= -8\Delta_{(IJK),(MNP)}\ ,\\
\mathrm{Tr}(\gamma^{IJKL}) &= 0,\\
\mathrm{Tr}(\gamma^{IJ}\gamma^{MN}) &= -8\Delta_{(IJ),(MN)}\ ,\\
\mathrm{Tr}(\gamma^{IJ}\gamma^{MNPQ}) &= 0,\\
\mathrm{Tr}(\gamma^{IJKL}\gamma^{MNPQ}) &= 8\Delta_{(IJKL),(MNPQ)} + 8\epsilon^{IJKLMNPQ}\ ,\\
\gamma^1\bar\gamma^2\cdots\bar\gamma^8 &= -\bar\gamma^1\gamma^2\cdots\gamma^8 = \mathbb{1}_8\ , && \text{(G.14)}
\end{aligned}$$

where $\Delta_{(I_1,\cdots I_n),(J_1,\cdots J_n)}$ is defined in eq.(E.4). The trace formulas also hold with γ and $\bar\gamma$ interchanged.

H Results of the Integrals

To compute the one-point functions in eqs.(3.35) and (E.9, E.10, E.11, E.12) one needs to evaluate certain integrals where various correlation functions on the full plane appear in the integrands. These correlation functions can be written in terms of various functions that have been computed in appendix F. Below we list the expected results for all the

required integrals containing some of these functions in the integrands. These results have been tested for various values of the positive integers m, n, p, q using Mathematica.

$$\oint_i \frac{dz}{2\pi i}\oint_{-i}\frac{dw}{2\pi i}\mathcal{J}_{m,n}(z,w)\tilde{F}^{(1,1)}(z,w,i,-i) = 8\delta_{mn} \ , \tag{H.15}$$

$$\oint_i \frac{dz}{2\pi i}\oint_{-i}\frac{dw}{2\pi i}\mathcal{J}_{m,n}(z,w)\tilde{F}^{(3,3)}(z,w,i,-i) = 8\delta_{mn} \ , \tag{H.16}$$

$$\oint_i \frac{dz_1 dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2 dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)F^{(1,1)}(z_1,w_1,z_2,w_2,i,-i) = 8(\delta_{mp}\delta_{nq}-\delta_{mq}\delta_{np}) \ , \tag{H.17}$$

$$\oint_i \frac{dz_1 dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2 dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)F_1^{(3,3)}(z_1,w_1,z_2,w_2,i,-i) = 8(\delta_{mp}\delta_{nq}+\delta_{mq}\delta_{np}) \ , \tag{H.18}$$

$$\oint_i \frac{dz_1 dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2 dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)F_2^{(3,3)}(z_1,w_1,z_2,w_2,i,-i) = 0 \ , \tag{H.19}$$

$$\oint_i \frac{dz_1 dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2 dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)F_2^{(3,3)}(z_1,w_1,z_2,w_2,-i,i) = 0 \ , \tag{H.20}$$

$$\oint_i \frac{dz_1 dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2 dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)F_1^{(5,5)}(z_1,w_1,z_2,w_2,i,-i) = 8(\delta_{mp}\delta_{nq}-\delta_{mq}\delta_{np}) \ , \tag{H.21}$$

$$\oint_i \frac{dz_1 dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2 dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)F_2^{(5,5)}(z_1,w_1,z_2,w_2,i,-i) = 0 \ . \tag{H.22}$$

$$\oint_i \frac{dz_1 dw_1}{(2\pi i)^2}\oint_{-i}\frac{dz_2 dw_2}{(2\pi i)^2}\mathcal{J}_{m,n,p,q}(z_1,w_1,z_2,w_2)F_2^{(5,5)}(z_1,w_1,z_2,w_2,-i,i) = 0 \ . \tag{H.23}$$

References

[1] J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond," http://www.slac.stanford.edu/spires/find/hep/www?irn=4634802

[2] N. Berkovits, JHEP **0004**, 018 (2000) [arXiv:hep-th/0001035]. N. Berkovits and B. C. Vallilo, JHEP **0007**, 015 (2000) [arXiv:hep-th/0004171]. N. Berkovits, JHEP **0009**, 046 (2000) [arXiv:hep-th/0006003]. N. Berkovits, Int. J. Mod. Phys. A **16**, 801 (2001) [arXiv:hep-th/0008145]. N. Berkovits, JHEP **0108**, 026 (2001) [arXiv:hep-th/0104247]. N. Berkovits, JHEP **0109**, 016 (2001) [arXiv:hep-th/0105050]. N. Berkovits and O. Chandia, Phys. Lett. B **514**, 394 (2001) [arXiv:hep-th/0105149]. N. Berkovits and P. S. Howe, Nucl. Phys. B **635**, 75 (2002) [arXiv:hep-th/0112160]. N. Berkovits, JHEP **0209**, 051 (2002) [arXiv:hep-th/0201151]. N. Berkovits, JHEP

0204, 037 (2002) [arXiv:hep-th/0203248]. N. Berkovits and O. Chandia, JHEP **0208**, 040 (2002) [arXiv:hep-th/0204121]. N. Berkovits and V. Pershin, JHEP **0301**, 023 (2003) [arXiv:hep-th/0205154]. N. Berkovits and J. Maldacena, JHEP **0210**, 059 (2002) [arXiv:hep-th/0208092]. N. Berkovits, arXiv:hep-th/0209059. N. Berkovits, JHEP **0210**, 071 (2002) [arXiv:hep-th/0210078]. N. Berkovits and N. Seiberg, JHEP **0307**, 010 (2003) [arXiv:hep-th/0306226]. N. Berkovits, JHEP **0409**, 047 (2004) [arXiv:hep-th/0406055]. N. Berkovits, arXiv:hep-th/0409159. N. Berkovits, arXiv:hep-th/0410079. N. Berkovits, arXiv:hep-th/0411170. N. Berkovits and D. Z. Marchioro, arXiv:hep-th/0412198.

[3] I. Oda and M. Tonin, Phys. Lett. B **520**, 398 (2001) [arXiv:hep-th/0109051]. P. A. Grassi, G. Policastro, M. Porrati and P. Van Nieuwenhuizen, JHEP **0210**, 054 (2002) [arXiv:hep-th/0112162]. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, JHEP **0211**, 001 (2002) [arXiv:hep-th/0202123]. G. Trivedi, Mod. Phys. Lett. A **17**, 2239 (2002) [arXiv:hep-th/0205217]. M. Matone, L. Mazzucato, I. Oda, D. Sorokin and M. Tonin, Nucl. Phys. B **639**, 182 (2002) [arXiv:hep-th/0206104]. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, Adv. Theor. Math. Phys. **7**, 499 (2003) [arXiv:hep-th/0206216]. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, Phys. Lett. B **553**, 96 (2003) [arXiv:hep-th/0209026]. B. C. Vallilo, JHEP **0212**, 042 (2002) [arXiv:hep-th/0210064]. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, arXiv:hep-th/0211095. M. Chesterman, JHEP **0402**, 011 (2004) [arXiv:hep-th/0212261]. Y. Aisaka and Y. Kazama, JHEP **0302**, 017 (2003) [arXiv:hep-th/0212316]. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, Class. Quant. Grav. **20**, S395 (2003) [arXiv:hep-th/0302147]. Y. Aisaka and Y. Kazama, JHEP **0308**, 047 (2003) [arXiv:hep-th/0305221]. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, Nucl. Phys. B **676**, 43 (2004) [arXiv:hep-th/0307056]. B. C. Vallilo, JHEP **0403**, 037 (2004) [arXiv:hep-th/0307018]. L. Anguelova and P. A. Grassi, JHEP **0311**, 010 (2003) [arXiv:hep-th/0307260]. O. Chandia and B. C. Vallilo, JHEP **0404**, 041 (2004) [arXiv:hep-th/0401226]. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, arXiv:hep-th/0402122. P. A. Grassi and P. van Nieuwenhuizen, Nucl. Phys. B **702**, 189 (2004) [arXiv:hep-th/0403209]. M. Chesterman, Nucl. Phys. B **703**, 400 (2004) [arXiv:hep-th/0404021]. Y. Aisaka and Y. Kazama, JHEP **0404**, 070 (2004) [arXiv:hep-th/0404141]. S. Guttenberg, J. Knapp and M. Kreuzer, JHEP **0406**, 030 (2004) [arXiv:hep-th/0405007]. P. A. Grassi and L. Tamassia,

JHEP **0407**, 071 (2004) [arXiv:hep-th/0405072]. P. A. Grassi and P. van Nieuwenhuizen, arXiv:hep-th/0408007. I. Oda and M. Tonin, Phys. Lett. B **606**, 218 (2005) [arXiv:hep-th/0409052].

[4] A. Sen, JHEP **9806**, 007 (1998) [arXiv:hep-th/9803194].

[5] A. Sen, JHEP **9808**, 010 (1998) [arXiv:hep-th/9805019].

[6] O. Bergman and M. R. Gaberdiel, Phys. Lett. B **441**, 133 (1998) [arXiv:hep-th/9806155].

[7] A. Sen, JHEP **9810**, 021 (1998) [arXiv:hep-th/9809111].

[8] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, Nucl. Phys. B **564**, 60 (2000) [arXiv:hep-th/9903123].

[9] A. Sen, arXiv:hep-th/9904207.

[10] A. Lerda and R. Russo, Int. J. Mod. Phys. A **15**, 771 (2000) [arXiv:hep-th/9905006]. J. H. Schwarz, arXiv:hep-th/9908144. M. R. Gaberdiel, Class. Quant. Grav. **17**, 3483 (2000) [arXiv:hep-th/0005029].

[11] J. Polchinski, arXiv:hep-th/9611050. J. Polchinski, S. Chaudhuri and C. V. Johnson, arXiv:hep-th/9602052. P. Di Vecchia and A. Liccardo, NATO Adv. Study Inst. Ser. C. Math. Phys. Sci. **556**, 1 (2000) [arXiv:hep-th/9912161]. P. Di Vecchia and A. Liccardo, arXiv:hep-th/9912275. I. V. Vancea, arXiv:hep-th/0109029. C. V. Johnson, arXiv:hep-th/0007170.

[12] M. B. Green, Phys. Lett. B **329**, 435 (1994) [arXiv:hep-th/9403040].

[13] M. B. Green and M. Gutperle, Nucl. Phys. B **476**, 484 (1996) [arXiv:hep-th/9604091].

[14] P. Mukhopadhyay, Nucl. Phys. B **600**, 285 (2001) [arXiv:hep-th/0011047].

[15] M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol. 1: Introduction," http://www.slac.stanford.edu/spires/find/hep/www?irn=1755021

[16] R. R. Metsaev, Nucl. Phys. B **625**, 70 (2002) [arXiv:hep-th/0112044]. R. R. Metsaev and A. A. Tseytlin, Phys. Rev. D **65**, 126004 (2002) [arXiv:hep-th/0202109].

[17] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, Class. Quant. Grav. **19**, L87 (2002) [arXiv:hep-th/0201081].

[18] D. Berenstein, J. M. Maldacena and H. Nastase, JHEP **0204**, 013 (2002) [arXiv:hep-th/0202021].

[19] J. M. Maldacena, arXiv:hep-th/0309246. A. Pankiewicz, Fortsch. Phys. **51**, 1139 (2003) [arXiv:hep-th/0307027]. J. C. Plefka, Fortsch. Phys. **52**, 264 (2004) [arXiv:hep-th/0307101]. D. Sadri and M. M. Sheikh-Jabbari, arXiv:hep-th/0310119. R. Russo and A. Tanzini, Class. Quant. Grav. **21**, S1265 (2004) [arXiv:hep-th/0401155].

[20] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200]. S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]. E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[21] P. Goddard and D. I. Olive, Int. J. Mod. Phys. A **1**, 303 (1986).

[22] N. Ishibashi, Mod. Phys. Lett. A **4**, 251 (1989).

[23] M. Kato and T. Okada, Nucl. Phys. B **499**, 583 (1997) [arXiv:hep-th/9612148].

[24] N. V. Suryanarayana - unpublished.

[25] A. Sen, JHEP **9910**, 008 (1999) [arXiv:hep-th/9909062].

[26] S. S. Gubser, A. Hashimoto, I. R. Klebanov and J. M. Maldacena, Nucl. Phys. B **472**, 231 (1996) [arXiv:hep-th/9601057]. M. R. Garousi and R. C. Myers, Nucl. Phys. B **475**, 193 (1996) [arXiv:hep-th/9603194].

[27] M. Billo and I. Pesando, background," Phys. Lett. B **536**, 121 (2002) [arXiv:hep-th/0203028].

[28] O. Bergman, M. R. Gaberdiel and M. B. Green, JHEP **0303**, 002 (2003) [arXiv:hep-th/0205183].

[29] M. R. Gaberdiel and M. B. Green, Annals Phys. **307**, 147 (2003) [arXiv:hep-th/0211122].

[30] V. A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, Nucl. Phys. B **288**, 173 (1987).